

EXAM FM QUESTIONS OF THE WEEK

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Week of December 12

Smith invests \$1,000 in a 5-year "step-up" Guaranteed Investment Certificate (GIC). The GIC will pay interest, compounded monthly, at the following annual rates:

Year 1: 3.0% , Year 2: 4.0% , Year 3: 6.0% , Year 4: 9.0% , Year 5: 12.0%

- (a) Find Smith's average nominal annual rate of return compounded monthly over the 5-year period.
- (b) One of the details of the GIC arrangement is that Smith can end the GIC early with a penalty. Suppose that Smith ends the GIC right at the beginning of the 5th year. Find Smith's average annual effective return in each of the following cases:
- (i) the penalty is 5% of the total interest earned on the investment up to the time it ended, and
- (ii) the penalty is 5% of the value of the investment at the time it is ended.

The solution can be found below.

Week of December 12 - Solution

(a) At the end of 5 years, the value of the GIC is

$$1000\left(1 + \frac{.03}{12}\right)^{12}\left(1 + \frac{.04}{12}\right)^{12}\left(1 + \frac{.06}{12}\right)^{12}\left(1 + \frac{.09}{12}\right)^{12}\left(1 + \frac{.12}{12}\right)^{12} = 1,403.28 .$$

This corresponds to an average monthly rate of return of j over the 5-year period, where

$(1 + j)^{60} = 1.40328$, so that $j = .005663$. This corresponds to a nominal annual rate of return compounded monthly of $12j = .0680$.

(b) The value of the investment before penalty at the beginning of the 5th year is

$$1000\left(1 + \frac{.03}{12}\right)^{12}\left(1 + \frac{.04}{12}\right)^{12}\left(1 + \frac{.06}{12}\right)^{12}\left(1 + \frac{.09}{12}\right)^{12} = 1,245.34 .$$

(i) The interest earned over the 4 years is 245.34, and 5% of that is a penalty of 12.27 .

The value of the GIC after penalty is $1,245.34 - 12.67 = 1,233.07$.

This corresponds to an average annual effective rate of return of i over the 4-year period, where

$$(1 + i)^4 = 1.23307 , \text{ so that } i = .0538 .$$

(ii) The penalty is $1,245.34 \times .05 = 62.27$, and the value of the GIC after penalty is

$$1,245.34 - 62.27 = 1,183.07 .$$

This corresponds to an average annual effective rate of return of i over the 4-year period, where

$$(1 + i)^4 = 1.18307 , \text{ so that } i = .0429 .$$