EXAM C QUESTIONS OF THE WEEK

S. Broverman, 2005

Week of December 12

You are given the following random sample of 5 losses from the random variable X:

2 , 2 , 3 , 5 , 6

The density function of X is to be estimated using kernel density estimation, where the kernel density function $k_y(x)$ is the pdf of the normal distribution with a mean of y and a variance of 1. Find the kernel smoothed estimate of the probability $P[2 < X \leq 3]$.

Solution can be found below.

Week of December 12 - Solution

The kernel smoothed estimate of the cdf is $\widehat{F}(x)$ is $\widehat{F}(x) = \sum_{\text{all } y_i} p(y_i) \cdot K_{y_i}(x)$

where the y_i's are the date points and $p(y_i)$ is the empirical probability at data point y_i, and $K_{y_i}(x)$ is the cdf for the normal distribution with mean y_i and variance 1.

From the given data, we have $p(2) = .4$, $p(3) = .2$, $p(5) = .2$, $p(6) = .2$.

The estimate of $P[2 < X \leq 3]$ is $\widehat{F}(3) - \widehat{F}(2)$.

$\widehat{F}(2) = (.4)K_2(2) + (.2)K_3(2) + (.2)K_5(2) + (.2)K_6(2)$.

$K_2(2) = \Phi(\frac{2-2}{1}) = .5$, $K_3(2) = \Phi(\frac{2-3}{1}) = .1587$,

$K_5(2) = \Phi(\frac{2-5}{1}) = .0013$, $K_6(2) = \Phi(\frac{2-6}{1}) = 0$,

so that $\widehat{F}(2) = (.4)(.5) + (.2)(.1587) + (.2)(.0013) + (.2)(0) = .232$.

$\widehat{F}(3) = (.4)K_2(3) + (.2)K_3(3) + (.2)K_5(3) + (.2)K_6(3)$.

$K_2(3) = \Phi(\frac{3-2}{1}) = .8413$, $K_3(3) = \Phi(\frac{3-3}{1}) = .5$,

$K_5(3) = \Phi(\frac{3-5}{1}) = .0228$, $K_6(2) = \Phi(\frac{3-6}{1}) = .0013$,

so that $\widehat{F}(3) = (.4)(.8413) + (.2)(.5) + (.2)(.0228) + (.2)(.0013) = .4413$.

The estimate of $P[2 < X \leq 3]$ is $.4413 - .232 = .2093$.