

## EXAM C QUESTIONS OF THE WEEK

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### Week of September 3/07

A random sample of size 4 has the following values 1, 2, 2, 3.

Kernel smoothing is applied to the data using a Pareto kernel. For sample point  $y$ , the kernel density function is Pareto with  $\alpha = 2$  and  $\theta = y$ . Find the kernel smoothed estimate of  $S(2)$ .

**The solution can be found below.**

## **Week of September 3/07 - Solution**

The Pareto survival function is  $S(x) = (\frac{\theta}{x+\theta})^\alpha$ .

With the given points, there is empirical probability of  $\frac{1}{4}$  at  $y = 1$  and  $3$ , and empirical probability of  $\frac{1}{2}$  at  $y = 2$ . With  $\alpha = 2$  and  $\theta = y$ , the kernel survival functions are:

for  $y = 1$ ,  $S_1(x) = (\frac{1}{x+1})^2$ , for  $y = 2$ ,  $S_2(x) = (\frac{2}{x+2})^2$ , and for  $y = 3$ ,  $S_3(x) = (\frac{3}{x+3})^2$ .

The kernel smoothed estimate of  $S(x)$  is

$$\hat{S}(x) = \frac{1}{4}S_1(x) + \frac{1}{2}S_2(x) + \frac{1}{4}S_3(x) = \frac{1}{4}(\frac{1}{x+1})^2 + \frac{1}{2}(\frac{2}{x+2})^2 + \frac{1}{4}(\frac{3}{x+3})^2.$$

$$\text{Then } \hat{S}(2) = \frac{1}{4}(\frac{1}{2+1})^2 + \frac{1}{2}(\frac{2}{2+2})^2 + \frac{1}{4}(\frac{3}{2+3})^2 = .2428.$$