EXAM C QUESTIONS OF THE WEEK

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Week of September 17/07

Product limit estimation is being applied to a data set which includes some right-censored and some left-truncated data. Successive death times are $y_1, y_2, y_{3,} ...$. The following estimated values are given:

Product limit estimate of $S(y_2)$ is 0.910000 ,

Product limit estimate of $S(y_3|X > y_1)$ is $.888889$,

Nelson-Aalen estimate of $H(y_2)$ is 0.091667 .

Nelson-Aalen estimate of $H(y_3|X > y_1)$ is 0.114286 .

Find the product limit estimate of $S(y_3)$.

The solution can be found below.

Week of September 17/07 - Solution

We are given

(i) $S_n(y_2) = (1 - \frac{s_1}{r_1})(1 - \frac{s_2}{r_2}) = 0.91$,

(ii) $S_n(y_3|X > y_1) = (1 - \frac{s_2}{r_2})(1 - \frac{s_3}{r_3}) = 0.888889$,

(iii) $\widehat{H}(y_2) = \frac{s_1}{r_1} + \frac{s_2}{r_2} = .091667$.

(iv) $\widehat{H}(y_3|X > y_1) = \frac{s_2}{r_2} + \frac{s_3}{r_3} = .114286$.

From (i) and (iii) , if we let $c = \frac{s_1}{r_1}$, we get $(1 - c)(1 - .091667 + c) = .91$.
This can be written in the form $c^2 - .091667c + .001667 = 0$.
Solving the resulting quadratic equation results in $c = .025$ or $.067$.
We do not have enough information to determine whether $\frac{s_1}{r_1} = .025$ and $\frac{s_2}{r_2} = .0667$ or $\frac{s_1}{r_1} = .0667$ and $\frac{s_2}{r_2} = .025$.

From (ii) and (iv), if we let $d = \frac{s_2}{r_2}$, we get $(1 - d)(1 - .114286 + d) = .888889$.
This can be written in the form $d^2 - .114286d + .003175 = 0$.
Solving the resulting quadratic equation results in $d = .0476$ or $.067$.
Since $d = \frac{s_2}{r_2}$ must be either .025 or .0666 from the previous paragraph, it must be the case that $\frac{s_2}{r_2} = .067$, and therefore, $\frac{s_1}{r_1} = .025$.

The product limit estimate of $S(y_3)$ is
$(1 - \frac{s_1}{r_1})(1 - \frac{s_2}{r_2})(1 - \frac{s_3}{r_3}) = (1 - .025)(.888889) = .867$.