

EXAM C QUESTIONS OF THE WEEK

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Week of October 29/07

The current price of a stock is 100. The stock price follows the lognormal stock price model. The stock has annual rate of return α , annual dividend rate δ and annual volatility σ (return and dividend are continuously compounded rates). The upper and lower limit of the 95% confidence interval for the stock price in one year are 160.64 and 67.81. If the annual dividend rate is doubled to 2δ , the upper and lower limit of the 95% confidence interval for the stock price in one year are 158.57 and 66.93. Find the expected value of the stock at the end of one year if the dividend is eliminated.

The solution can be found below.

Week of October 29/07 - Solution

Based on dividend rate δ , the upper and lower limits of the 95% confidence interval for the stock price after one year are $100e^{(\alpha-\delta-.5\sigma^2)+1.96\sigma}$ and $100e^{(\alpha-\delta-.5\sigma^2)-1.96\sigma}$, and these are 160.64 and 67.81 .

Based on dividend rate 2δ , the upper and lower limits of the 95% confidence interval for the stock price after one year are $S_0e^{(\alpha-2\delta-.5\sigma^2)+1.96\sigma}$ and $S_0e^{(\alpha-2\delta-.5\sigma^2)-1.96\sigma}$, and these are 156.62 and 66.07 .

From information about the interval based on dividend rate δ we get

$$\ln 100 + \alpha - \delta - .5\sigma^2 + 1.96\sigma = \ln 160.64 \text{ and}$$

$$\ln 100 + \alpha - \delta - .5\sigma^2 - 1.96\sigma = \ln 67.81 .$$

It follows that $\alpha - \delta - .5\sigma^2 = .0428$, and $1.96\sigma = .4312$.

From information about the interval based on dividend rate 2δ we get

$$\ln 100 + \alpha - 2\delta - .5\sigma^2 + 1.96\sigma = \ln 158.57 .$$

Therefore, $\delta = \ln 160.64 - \ln 158.57 = .0130$.

Then, $\alpha = .08$. If the dividend is eliminated, the expected stock price after one year is $100e^{.08} = 108.33$.