

EXAM C QUESTIONS OF THE WEEK

S. Broverman, 2007

Week of October 22/07

The loss random variable L is a mixture of two exponential distributions:

- mean 1 with mixing weight .25, and
- mean 2 with mixing weight .75.

The following distortion function is applied to $S(x)$, the survival function of the mixed distribution: $g(x) = \begin{cases} 0 & \text{if } 0 \leq x < .9 \\ 1 & \text{if } .9 \leq x \leq 1 \end{cases}$.

Find the risk measure $\int_0^\infty g(S(x)) dx$.

The solution can be found below.

Week of October 22/07 - Solution

For this distortion function, the risk measure is the .9-VaR of X , which is the 90th percentile of the mixed distribution. The 90th percentile of X is c , where $S(c) = .1$, so that

$.25e^{-c} + .75e^{-c/2} = .1$. With the substitution $u = e^{-c/2}$, this equation becomes $5u^2 + 15u - 2 = 0$. Solving for u results in $u = .1279$ or -3.1279 .

We ignore the negative root, since $u = e^{-c/2} > 0$.

Then solving for c from $e^{-c/2} = .1279$ results in $c = 4.11$, the 90th percentile of the mixed distribution. This is the value of the distortion risk measure.