EXAM C QUESTION OF THE WEEK

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Week of May 5/08

The prior distribution of λ is a gamma distribution with parameters $\alpha = 3$ and $\theta = 2$.
The conditional distribution of X given λ is Poisson with a mean of λ.

The unconditional distribution of X is to be simulated in two steps.
Step 1: simulate a value of λ
Step 2: simulate a value of X given the value of λ simulated in Step 1.
To apply step 1, we use the fact that the gamma distribution with parameters $\alpha = 3$ and θ is the sum of three independent exponential random variables each with mean θ, and simulate three independent exponentials using the inverse transformation method and add the simulated values to get the simulated gamma distribution value. To apply step 2 we use the product algorithm for the Poisson.

The sequence of uniform $(0, 1)$ numbers to be used in the overall simulation are

$.2$, $.6$, $.3$, $.1$, $.7$, $.7$, $.1$

These numbers are used in the order given, with the first three used to simulate the gamma distribution and the remaining numbers used in step 2. Each number is used once until the simulation is complete.

Find the value of X simulated.

The solution can be found below.

Week of May 5/08 - Solution

The simulation of an exponential variable Y with mean θ using the inverse transform method is $y = -\theta\, ln(1-u)$. In this case, $\theta = 1$, so the three simulated exponential values are
$Y_1 = -2\, ln(1-.2) = .446287$, $Y_2 = -2\, ln(1-.6) = 1.832581$, and
Y_3= $-2\, ln(1-.3) = .713350$.
The simulated value of the gamma is $\lambda = Y_1 + Y_2 + Y_3 = 2.992$.
This completes Step 1.
According to the product algorithm we multiply the successive uniform $(0,1)$ values until the product is first less then $e^{-2.992} = .0502$. The remaining uniform $(0,1)$ numbers are
.1 , .7 , .7 , .1 . We see that $.1 \times .7 > .05052 > .1 \times .7 \times .7$.
Since the product of the first two uniform numbers was greater than $e^{-2.992}$, but the product of the first three was less than $e^{-2.992}$, the simulated value of X is 2.