

EXAM C QUESTIONS OF THE WEEK

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Week of May 14/07

You are given a random sample of 3 values from a distribution F : 4, 5, 9

You estimate the median of X using the estimator $\frac{1}{2}(\text{smallest } X_i + \text{largest } X_i)$.

Determine the bootstrap approximation to the mean square error.

- A) 2.2 B) 2.4 C) 2.6 D) 2.8 E) 3.0

The solution can be found below.

Week of May 14/07 - Solution

MSE is approximated (estimated) by using the corresponding quantity in the empirical distribution. The empirical distribution consists of the three data points, so the median of the empirical distribution is 5. The mean square error of the estimator is $E[(\hat{\theta} - 5)^2]$. Since the empirical distribution consists of three points, there are $3^3 = 27$ possible samples of size 3 that can be drawn from the empirical distribution, each with probability $\frac{1}{27}$. The samples and values of the estimator $\hat{\theta}$ are

Sample #	1	2	3	4	5	6	7	8	9
Sample	4,4,4	4,4,5	4,4,9	4,5,4	4,5,5	4,5,9	4,9,4	4,9,5	4,9,9
$\hat{\theta} =$ Sample median	4	4.5	6.5	4.5	4.5	6.5	6.5	6.5	6.5

Sample #	10	11	12	13	14	15	16	17	18
Sample	5,4,4	5,4,5	5,4,9	5,5,4	5,5,5	5,5,9	5,9,4	5,9,5	5,9,9
Sample median	4.5	4.5	6.5	4.5	5	7	6.5	7	7

Sample #	19	20	21	22	23	24	25	26	27
Sample	9,4,4	9,4,5	9,4,9	9,5,4	9,5,5	9,5,9	9,9,4	9,9,5	9,9,9
Sample median	6.5	6.5	6.5	6.5	7	7	6.5	7	9

$$MSE = \frac{1}{27}(4 - 5)^2 + \frac{6}{27}(4.5 - 5)^2 + \frac{1}{27}(5 - 5)^2 + \frac{12}{27}(6.5 - 5)^2 + \frac{6}{27}(7 - 5)^2 + \frac{1}{27}(9 - 5)^2 = 2.57. \quad \text{Answer: C}$$