EXAM C QUESTION OF THE WEEK

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Week of May 12/08

The distribution of X is a mixture of two continuous random variables. The mixing weight is a for random variable X_1 and the mixing weight is $1-a$ for random variable X_2, where $0<a<1$. Suppose that $0<\alpha<1$, and Q_α is the 100α-th percentile of the mixed distribution X.

$ECP^F_{X_1,d}$ and $ECP^F_{X_2,d}$ denote the expected cost per payment for random variables X_1 and X_2 with franchise deductible d, respectively. Show that CTE_α for the mixture distribution is the weighted average of $ECP^F_{X_1,Q_\alpha}$ and $ECP^F_{X_2,Q_\alpha}$ using the mixing weights a and $1-a$.

The solution can be found below.

Week of May 12/08 - Solution

$CTE_\alpha = ECP^F$ for a franchise deductible of Q_α for the mixture distribution.
This is $\frac{E[(X-Q_\alpha)_+]}{1-\alpha} + Q_\alpha$.
But, $E[(X-d)_+] = \int_d^\infty [1 - F_X(t)]\,dt$
$= a \cdot \int_d^\infty [1 - F_{X_1}(t)]\,dt + (1-a) \cdot \int_d^\infty [1 - F_{X_2}(t)]\,dt$
$= a \cdot E[(X_1 - d)_+] + (1-a) \cdot E[(X_2 - d)_+]$
Therefore
$ECP^F_{X,Q_\alpha} = \frac{E[(X-Q_\alpha)_+]}{1-\alpha} = a \cdot \frac{E[(X_1-Q_\alpha)_+]}{1-\alpha} + (1-a) \cdot \frac{E[(X_2-Q_\alpha)_+]}{1-\alpha}$
$= a \cdot ECP^F_{X_1,Q_\alpha} + (1-a) \cdot ECP^F_{X_2,Q_\alpha}$
$CTE_\alpha = \frac{E[(X-Q_\alpha)_+]}{1-\alpha} + Q_\alpha$
$= a \cdot ECP^F_{X_1,Q_\alpha} + (1-a) \cdot ECP^F_{X_2,Q_\alpha} + a \cdot Q_\alpha + (1-a) \cdot Q_\alpha$
$= a \cdot [ECP^F_{X_1,Q_\alpha} + Q_\alpha] + (1-a) \cdot [ECP^F_{X_2,Q_\alpha} + Q_\alpha]$
This is the weighted average of the expected cost per loss for franchise deductible Q_α for X_1 and X_2.