EXAM C QUESTIONS OF THE WEEK

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Week of March 5/07

Let $X_1, \ldots, X_n$ be a random sample from a distribution with density function

$$f(x;\theta) = \begin{cases} (1-\theta)\theta^x, \; x=0,1,2,\ldots;\, 0\le\theta<1 \\ \\ 0, \text{ elsewhere} \end{cases}.$$

What is the maximum likelihood estimator for θ?

A) $\bar{X}$ B) $1+\bar{X}$ C) $3\bar{X}$ D) $\frac{\bar{X}}{1+\bar{X}}$ E) $1+\frac{1}{\bar{X}}$

The solution can be found below.

Week of March 5/07 - Solution

The likelihood function for the random sample of size n is $L(x_1, ...x_n; \theta) = f(x_1; \theta) \cdot f(x_2; \theta) \cdots f(x_n: \theta) = (1-\theta)^n \cdot \theta^{\Sigma x_i}$. To maximize L with respect to θ we differentiate L with respect to θ and set equal to 0:

$\frac{dL}{d\theta} = -n \cdot (1-\theta)^{n-1} \cdot \theta^{\Sigma x_i} + \left[\sum_{i=1}^{n} x_i\right] \cdot (1-\theta)^n \cdot \theta^{[\Sigma x_i]-1} = 0$, or equivalently,

$-n\theta + \left[\sum_{i=1}^{n} x_i\right] \cdot (1-\theta) = 0$, or equivalently, $\theta = \frac{\Sigma x_i}{[\Sigma x_i]+n} = \frac{\bar{X}}{\bar{X}+1}$. That this is the value of θ that maximizes L can be verified by the second derivative test. Note that it would have been somewhat more efficient to maximize $\ln(L)$. Answer: D