

# EXAM C QUESTIONS OF THE WEEK

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## Week of March 26/07

A random sample of 1000 observations from a loss distribution has been grouped into five intervals as follows:

<u>Interval</u>	<u>Number of Observations</u>
[0, 3.0)	180
[3.0, 7.5)	180
[7.5, 15.0)	235
[15.0, 40.0)	255
[40.0, $\infty$ )	150

A Pareto distribution is fit to the data using the minimum chi-square estimator. The estimated parameter values are  $\alpha = 3.5$  and  $\theta = 50$  ( $F(x) = 1 - (\frac{50}{x+50})^{3.5}$ ).

Find the value of the chi-square goodness-of-fit statistic for testing the fit of the model to the data.

- A) Less than 9.0      B) At least 9.0, but less than 9.2  
C) At least 9.2, but less than 9.4      D) At least 9.4, but less than 9.6  
E) At least 9.6

**The solution can be found below.**

## **Week of March 26/07 - Solution**

$$\chi^2 = \sum_{j=1}^k \frac{(O_j - E_j)^2}{E_j} . O_1 = 180, O_2 = 180, O_3 = 235, O_4 = 255, O_5 = 150$$

$$E_1 = 1000F(3) = 1000[1 - (\frac{50}{53})^{3.5}] = 184.5,$$

$$E_2 = 1000[F(7.5) - F(3)] = 1000[(\frac{50}{53})^{3.5} - (\frac{50}{57.5})^{3.5}] = 202.4,$$

$$E_3 = 1000[F(15) - F(7.5)] = 1000[(\frac{50}{57.5})^{3.5} - (\frac{50}{65})^{3.5}] = 213.9,$$

$$E_4 = 1000[F(40) - F(15)] = 1000[(\frac{50}{65})^{3.5} - (\frac{50}{90})^{3.5}] = 271.4,$$

$$E_5 = 1000[F(\infty) - F(40)] = 1000[(\frac{50}{90})^{3.5}] = 127.8.,$$

$$\chi^2 = \frac{(180-184.5)^2}{184.5} + \frac{(180-202.4)^2}{202.4} + \frac{(235-213.9)^2}{213.9} \\ + \frac{(255-271.4)^2}{271.4} + \frac{(150-127.8)^2}{127.8} = 9.5. \quad \text{Answer: D}$$