

EXAM C QUESTION OF THE WEEK

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Week of March 17/08

The product-limit estimate is used to estimate survival probabilities.

Analysis of the data shows that there is unclear information regarding one of the subjects in the study. It is not clear if the subject is to be recorded as a death at the first death point or as being right-censored at the first death point.

- the product limit estimate of $S(y_1)$ (survival probability to the first death point) increases by a ratio of 1.012987 if the subject is regarded as a right-censoring rather than as a death at the first death point y_1
- the Nelson-Aalen estimate of $H(y_2)$ increases by a ratio of 1.070825 if the subject is regarded as a death rather than as a right-censoring at the first death point
- the Nelson-Aalen estimate increases by .011765 if the subject is regarded as a death rather than as a right-censoring at the first death point

Determine the product-limit estimate of $S(y_2)$, the survival probability to the second death point, assuming that the uncertain subject is a right-censored.

The solution can be found below.

Week of March 17/08 - Solution

Let s_1 denote the number of deaths at time y_1 , not including the uncertain subject.

We are given $\frac{1 - \frac{s_1}{r_1}}{1 - \frac{s_1 + 1}{r_1}} = 1.012987$ (1) , $\frac{\frac{s_1 + 1}{r_1} + \frac{s_2}{r_2}}{\frac{s_1}{r_1} + \frac{s_2}{r_2}} = 1.063167$ (2) and

$$\frac{s_1 + 1}{r_1} + \frac{s_2}{r_2} - \left(\frac{s_1}{r_1} + \frac{s_2}{r_2} \right) = .011765 \text{ (3)}.$$

From the third equation we get $\frac{1}{r_1} = .011765$.

Applying this to the first equation, we get $\frac{1 - \frac{s_1}{r_1}}{1 - \frac{s_1 + 1}{r_1}} = \frac{1 - \frac{s_1}{r_1}}{1 - (\frac{s_1}{r_1} + .011765)} = 1.012987$

and it follows that $\frac{s_1}{r_1} = .082353$.

Applying $\frac{1}{r_1} = .011765$ and $\frac{s_1}{r_1} = .082353$ to the second equation results in $\frac{s_2}{r_2} = .103895$.

The product-limit estimate of $S(y_2)$ is

$$\left(1 - \frac{s_1}{r_1}\right)\left(1 - \frac{s_2}{r_2}\right) = (1 - .082353)(1 - .103895) = .8223 .$$