EXAM C QUESTIONS OF THE WEEK

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Week of March 12/07

The following random sample of size 2 is taken from a Pareto distribution:
18.7 , 51.5
An iterative maximum likelihood estimation method is applied to estimate α and θ.
A starting estimate of θ is $\theta_1 = 100$. α is estimated using maximum likelihood estimation.
The value of α found is denoted α_1 and it used to find the mle of θ, denoted θ_2.
The value of θ_2 is used to find the next mle of α denoted α_2 .
Find α_2 .

The solution can be found below.

Week of March 12/07 - Solution

The log of the density of the Pareto distribution is
$ln\, f(x) = ln\, \alpha + \alpha\, ln\, \theta - (\alpha+1)\, ln(x+\theta)$ and

$\frac{\partial}{\partial\theta}\, ln\, f(x) = \frac{\alpha}{\theta} - \frac{\alpha+1}{x+\theta}$, $\frac{\partial}{\partial\alpha}\, ln\, f(x) = \frac{1}{\alpha} + ln\, \theta - ln(x+\theta)$.

Given $\theta_1 = 100$, with 2 sample values, to find the mle of α, we solve
$\frac{\partial}{\partial\alpha}\, ln\, L = \frac{2}{\alpha} + 2\, ln\, \theta - \Sigma ln(x_i+\theta) = \frac{2}{\alpha} + 2\, ln\, 100 - (ln\, 118.7 + ln\, 151.5) = 0$.
Solving for α results in $\alpha_1 = 3.41$.

Now, with $\alpha_1 = 3.41$, to find the mle of θ, we solve
$\frac{\partial}{\partial\theta}\, ln\, L = \frac{2\alpha}{\theta} - \Sigma \frac{\alpha+1}{x_i+\theta} = \frac{2(3.41)}{\theta} - \frac{4.41}{18.7+\theta} - \frac{4.41}{51.5+\theta} = 0$.
This expression becomes $\frac{6.82(18.7+\theta)(51.5+\theta) - 4.41\theta(51.5+\theta) - 4.41\theta(18.7+\theta)}{\theta(18.7+\theta)(51.5+\theta)} = 0$.
The numerator is $6568.001 + 169.182\theta - 2\theta^2$.
Setting this equal to 0 results in two solutions for θ, 113.5 and -28.9 .
We ignore the negative root and use $\theta_2 = 113.5$ as the new estimate for θ.

The next estimate of α is found from
$\frac{2}{\alpha} + 2\, ln\, 113.5 - (ln\, 132.2 + ln\, 165.0) = 0$.
Solving for α results in $\alpha_2 = 3.80$.