EXAM C QUESTIONS OF THE WEEK

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Week of January 7/08

Q has a beta $a, b, 1$ distribution ($\theta = 1$, Q is distributed on the interval $(0, 1)$).

The conditional distribution of Y given $Q = q$ has probability function
$P(Y = k|Q = q) = \frac{q^k}{(1+q)^{k+1}}$.

You are given that the unconditional mean and variance of Y are
$E(Y) = .6$ and $Var(Y) = 1.04$.

Find the values of a and b.

The solution can be found below.

Week of January 7/08 - Solution

The conditional distribution of Y given Q has a geometric distribution with mean Q and $E(Y|Q=q)=q$, $Var(Y|Q=q)=q(1+q)$.

The first and second moments of Q are $E(Q)=\frac{a}{a+b}$ and $E(Q^2)=\frac{(a+1)\times a}{(a+b+1)\times(a+b)}$.

Y has a continuous mixture distribution. The unconditional mean of Y is
$E(Y)=E[E(Y|Q)]=E[Q]=\frac{a}{a+b}=.6$.
The unconditional variance of Y is
$Var(Y)=Var[E(Y|Q)]+E[Var(Y|Q)]=Var(Q)+E[Q(1+Q)]$
$=E(Q^2)-[E(Q)]^2+E(Q)+E(Q^2)=2E(Q^2)+E(Q)-[E(Q)]^2$
$=\frac{2(a+1)\times a}{(a+b+1)\times(a+b)}+\frac{a}{a+b}-(\frac{a}{a+b})^2=\frac{2(a+1)}{(a+b+1)}\times .6+.6-(.6)^2$
$=\frac{1.2(a+1)}{(a+b+1)}+.24=1.04$, and then $\frac{a+1}{a+b+1}=\frac{2}{3}$.

From the two equations $\frac{a}{a+b}=.6$ and $\frac{a+1}{a+b+1}=\frac{2}{3}$,
we get $a=.6a+.6b$ and $3a+3=2a+2b+2$.
Solving these equations results in $a=3$ and $b=2$.