EXAM C QUESTIONS OF THE WEEK

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When an ordinary deductible d is applied to a loss random variable X, we can formulate the distribution of Y_P, the cost per payment random variable. Y_P is the conditional distribution of $X-d$ given that $X>d$.

For each of the following distributions for X, and for ordinary deductible $d>0$, we wish to investigate the relative tail weights of Y_P and X.

I. exponential with mean $\theta>0$

II. Weibull with parameters $\tau>1$ and $\theta>0$

III. Pareto with parameters α and θ

For which of these distributions does Y_P have lighter right tails than X?

The solution can be found below.

Week of January 28/08 - Solution

We say that Y has a lighter right tail than X is $\lim_{x\to\infty} \frac{S_Y(x)}{S_X(x)} = 0$.

I. If X has an exponential distribution with mean θ, then Y_P also has an exponential distribution with mean θ. Therefore, X and Y_P have proportional tail weights. Y_P does not have a lighter right tail than X.

II. If X has a Weibull distribution, then $S_X(x) = e^{-(x/\theta)^\tau}$.
The survival function for Y_P is
$S_{Y_P}(x) = P(Y_P > x) = P(X - d > x | X > d) = P(X > d + x | X > d)$
$= \frac{P(X > d+x)}{P(X > d)} = \frac{e^{-[(d+x)/\theta]^\tau}}{e^{-(d/\theta)^\tau}}$.
Then $\frac{S_{Y_P}(x)}{S_X(x)} = \frac{e^{-[(d+x)/\theta]^\tau}}{e^{-(d/\theta)^\tau}} \Big/ e^{-(x/\theta)^\tau} = \frac{e^{-[(d+x)/\theta]^\tau}}{e^{-(d/\theta)^\tau} e^{-(x/\theta)^\tau}} = e^{-\frac{1}{\theta^\tau}[(d+x)^\tau - d^\tau - x^\tau]}$.
Since $\tau > 1$, it follows that $(d+x)^\tau - d^\tau - x^\tau \to \infty$ as $x \to \infty$.
Therefore, $\lim_{x\to\infty} \frac{S_{Y_P}(x)}{S_X(x)} = 0$, so Y_P does have a lighter right tail than X.

III. If X is Pareto with parameters α and θ, then $S_X(x) = (\frac{\theta}{x+\theta})^\alpha$.
Also, in for this X, the distribution of Y_P is also Pareto, but with parameters α and $\theta + d$. Therefore, the survival function for Y_P is $S_{Y_P}(x) = (\frac{\theta+d}{x+\theta+d})^\alpha$.
Then $\frac{S_{Y_P}(x)}{S_X(x)} = (\frac{\theta+d}{x+\theta+d})^\alpha \Big/ (\frac{\theta}{x+\theta})^\alpha = (\frac{\theta+d}{\theta})^\alpha \times (\frac{x+\theta}{x+\theta+d})^\alpha$,
and $\lim_{x\to\infty} \frac{S_{Y_P}(x)}{S_X(x)} = (\frac{\theta+d}{\theta})^\alpha$. Since this is between 0 and ∞, X and Y_P have proportional right tail weights.