EXAM C QUESTIONS OF THE WEEK

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Week of February 11/08

S has a compound distribution.

The frequency N is Poisson with a mean of 1.

The severity random variable X has a distribution for which $X-1$ has a Poisson distribution with mean 1.

Frequency and severity are independent, and severity amounts are independent of one another.

$P = P(S < 4)$ exactly
and
$Q = P(S < 4)$ using the normal approximation to S (with continuity correction).

Find Q/P .

The solution can be found below.

Week of February 11/08 - Solution

$P(S=0)=P(N=0)=e^{-1}$.

$P(S=1)=P(N=1)\times P(X=1)=P(N=1)\times P(X-1=0)=e^{-1}\times e^{-1}=e^{-2}$.

$P(S=2)=P(N=1)\times P(X=2)+P(N=2)\times[P(X=1)]^2$
$=P(N=1)\times P(X-1=1)+P(N=2)\times[P(X-1=0)]^2$
$=e^{-1}\times e^{-1}+\frac{e^{-1}}{2!}\times(e^{-1})^2=e^{-2}+\frac{e^{-3}}{2}$.

$P(S=3)=P(N=1)\times P(X=3)+P(N=2)\times P(X=1)\times P(X=2)\times 2$
$+P(N=3)\times[P(X=1)]^3$
$=P(N=1)\times P(X-1=2)+P(N=2)\times P(X-1=0)\times P(X-1=1)\times$
$+P(N=3)\times[P(X-1=0)]^3$
$=e^{-1}\times\frac{e^{-1}}{2!}+\frac{e^{-1}}{2!}\times e^{-1}\times e^{-1}\times 2+\frac{e^{-1}}{3!}\times[e^{-1}]^3$
$=\frac{e^{-2}}{2}+e^{-3}+\frac{e^{-4}}{6}$.

The exact probability $P(S<4)$ is $P(S=0)+P(S=1)+P(S=2)+P(S=3)$
$P=e^{-1}+e^{-2}+e^{-2}+\frac{e^{-3}}{2}+\frac{e^{-2}}{2}+e^{-3}+\frac{e^{-4}}{6}$
$=e^{-1}+\frac{5e^{-2}}{2}+\frac{3e^{-2}}{2}+\frac{e^{-4}}{6}=.783951$.

The mean and variance of N are both 1, and the mean of X is 2 and the variance of X is 1.

The mean and variance of S are $E(S)=E(N)\cdot E(X)=(1)(2)=2$
and
$Var(S)=E(N)\times E(X^2)=E(X)\times[Var(X)+[E(X)]^2]=(1)[1+2^2]=5$.

Applying the normal approximation (with continuity correction) to $P(S<4)$ results in
$Q=P(S\leq 3.5)=P\left(\frac{S-E(S)}{\sqrt{Var(S)}}\leq\frac{3.5-E(S)}{\sqrt{Var(S)}}\right)=\Phi\left(\frac{3.5-2}{\sqrt{5}}\right)=\Phi(.6708)=.7486$
(this is $\Phi(.67)$) .

$Q/P=\frac{.7486}{.7840}=.955$.