EXAM C QUESTIONS OF THE WEEK

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Week of August 13/07

A compound distribution S has frequency N and severity X, both of which are members of the $(a, b, 0)$ class. You are given the following:
$E(N) = 2.4$, $Var(N) = 1.92$, $E(S) = 14.4$, $Var(S) = 126.72$

Find $P(S = 0)$.

The solution can be found below.

Week of August 13/07 - Solution

The probability generating functions of S, N and X satisfy the relationship
$P_S(t) = P_N(P_X(t))$. Then, $P(S = 0) = P_S(0) = P_N(P_X(0))$.

An $(a, b, 0)$ distribution must be either Poisson, Negative Binomial or Binomial.
Binomial is the only one of the three with expected value greater than variance.
Therefore, N is binomial, say with parameters q and m.
$E[N] = mq = 2.4$ and $Var[N] = mq(1 - q) = 1.92$,
and it follows that $q = .2$ and $m = 12$. The probability generating function of N is
$P_N(z) = [1 + q(z - 1)]^m = [1 + (.2)(z - 1)]^{12}$.

The mean of S is $E[S] = E[N] \cdot E[X]$, so that $14.4 = 2.4E[X]$
and we get $E[X] = 6.0$.

The variance of S is $Var[S] = E[N] \cdot Var[X] + Var[N] \cdot (E[X])^2$,
so that $126.72 = 2.4Var[X] + 1.92(6.0)^2$, and we get $Var[X] = 24$.

Since X is an $(a, b, 0)$ distribution, it is either Poisson, Negative Binomial or Binomial.
The Negative Binomial distribution is the only one of the three whose mean is less than its
variance. Therefore X has a negative binomial distribution, say with parameters r and β.
$E[X] = r\beta = 6$ and $Var[X] = r\beta(1 + \beta) = 24$, and it follows that $\beta = 3$ and $r = 2$.
The probability generating function of X is $P_X(t) = \frac{1}{[1-\beta(t-1)]^r} = \frac{1}{[1-3(t-1)]^2}$.

Then, $P_X(0) = \frac{1}{[1-3(-1)]^2} = \frac{1}{16}$, and
$P(S = 0) = P_S(0) = P_N(P_X(0)) = P_N(\frac{1}{16}) = [1 + (.2)(\frac{1}{16} - 1)]^{12} = .0828$.