EXAM C QUESTION OF THE WEEK

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Week of April 28/08

A compound distribution S consists of negative binomial frequency N with parameters r and β, and exponential severity with mean θ. Limited fluctuation credibility is applied to N, X and S using the 5% and .90 probability factors. Recall that the negative binomial random variable has mean $r\beta$ and variance $r\beta(1+\beta)$. You are given the following information:

- if credibility is applied to N, and 300 exposures of N are available,
the partial credibility factor Z is .832049

- if credibility is applied to X, and the sum of the observed values of X is 1,000,000,
the partial credibility factor is .607906

- if credibility is applied to S, and 400 exposures of S are available,
the partial credibility factor is .889883

If credibility is applied to S and the sum of the observed values is 12,000,
find the partial credibility factor.

The solution can be found below.

Week of April 28/08 - Solution

Full credibility standard for N based on number of exposures is
$1082.4 \cdot \frac{V(N)}{[E(N)]^2} = 1082.4 \cdot \frac{r\beta(1+\beta)}{(r\beta)^2} = 1082.4 \cdot \frac{1+\beta}{r\beta}$.
We are given that $\sqrt{300 \Big/ (1082.4 \cdot \frac{1+\beta}{r\beta})} = .832049$ from which we get $\frac{1+\beta}{r\beta} = .400$

Full credibility standard for X based on sum of observed values is
$1082.4 \cdot \frac{V(X)}{E(X)} = 1082.4 \cdot \frac{\theta^2}{\theta} = 1082.4\theta$.
We are given that $\sqrt{1,000,000 \Big/ (1082.4\theta)} = .607906$ from which we get $\theta = 2500.0$.

Full credibility standard for S based on number of exposures is
$1082.4 \cdot \frac{V(S)}{[E(S)]^2} = 1082.4 \cdot \frac{r\beta\theta^2 + r\beta(1+\beta)\theta^2}{(r\beta\theta)^2} = 1082.4 \cdot \frac{2+\beta}{r\beta}$.
We are given that $\sqrt{400 \Big/ (1082.4 \cdot \frac{2+\beta}{r\beta})} = .889883$ from which we get $\frac{2+\beta}{r\beta} = .46667$.

Using the equations $\frac{1+\beta}{r\beta} = .400$ and $\frac{2+\beta}{r\beta} = .46667$, we get $\beta = 5$ and $r = 3$.

The full credibility standard applied to S based on the sum of observed values has a standard of
$1082.4 \cdot \frac{V(S)}{E(S)} = 1082.4 \cdot \frac{r\beta\theta^2 + r\beta(1+\beta)\theta^2}{r\beta\theta} = 1082.4[\theta + (1+\beta)\theta] = 18,942,000$.
With a sum of observed values of 12,000,000 , the partial credibility factor
is $\sqrt{\frac{12,000,000}{18,942,000}} = .796$.