

EXAM C QUESTIONS OF THE WEEK

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Week of April 23/07

A portfolio of insurance policies consists of three types of policies. The distribution of the number of losses in one year for each type of policy is summarized as follows:

Policy Type	Type I	Type II	Type III
Annual Number of Losses	Poisson With Mean 1	Poisson With Mean 2	Poisson With Mean 4

Half of the policies are of Type I, one-quarter of the policies are of Type II and one-quarter are Type III.

A policy is chosen at random, and the number of losses in one year is 1.

Find the Bayesian premium (the expected number of claims next year for the same policy).

The solution can be found below.

Week of April 23/07 - Solution

Type I, $\frac{1}{2}$

$$P(X = 1 | \text{Type I}) = e^{-1}$$

Type II, $\frac{1}{4}$

$$P(X = 1 | \text{Type II}) = 2e^{-2}$$

Type III, $\frac{1}{4}$

$$P(X = 1 | \text{Type III}) = 4e^{-4}$$

$$P(X = 1 \cap \text{Type I})$$

$$= \frac{1}{2}e^{-1}$$

$$P(X = 1 \cap \text{Type II})$$

$$= \frac{1}{2}e^{-2}$$

$$P(X = 1 \cap \text{Type III})$$

$$= e^{-4}$$

$$P(X = 1) = \frac{1}{2}e^{-1} + \frac{1}{2}e^{-2} + e^{-4}$$

$$P(\text{Type I} | X = 1) = \frac{\frac{1}{2}e^{-1}}{\frac{1}{2}e^{-1} + \frac{1}{2}e^{-2} + e^{-4}} = .681 ,$$

$$P(\text{Type II} | X = 1) = \frac{\frac{1}{2}e^{-2}}{\frac{1}{2}e^{-1} + \frac{1}{2}e^{-2} + e^{-4}} = .251 ,$$

$$P(\text{Type III} | X = 1) = \frac{e^{-4}}{\frac{1}{2}e^{-1} + \frac{1}{2}e^{-2} + e^{-4}} = .068 .$$

$$E[X_2 | X_1 = 1] = E[X_2 | \text{Type I}] \cdot P(\text{Type I} | X_1 = 1) + E[X_2 | \text{Type II}] \cdot P(\text{Type II} | X_1 = 1)$$

$$+ E[X_2 | \text{Type III}] \cdot P(\text{Type III} | X_1 = 1)$$

$$= (1)(.681) + (2)(.251) + (4)(.068) = 1.455 .$$