EXAM C QUESTIONS OF THE WEEK

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Week of April 2/07

Questions 1 to 4 are based on a survival study of 10 patients who have just been diagnosed with some medical condition. They are divided into 2 groups of 5 each. The effect of a medical treatment is being considered. Members of Group I do not receive the treatment $(Z = 0)$ and members of Group II receive the treatment $(Z = 1)$. Over the first two time units, there is one death from Group 1 at time 1 and one death at time 2, and from Group 2 there is one death at time 2, at which time, the study ends. The Cox proportional hazard model with parameter β is used to formulate partial likelihood.

Find the maximum likelihood estimate of β.
A) $-.80$ B) $-.82$ C) $-.84$ D) $-.86$ E) $-.88$

The solution can be found below.

Week of April 2/07 - Solution

$L(\beta) = \frac{1}{[5+5e^{\beta}]^1} \cdot \frac{e^{\beta}}{[4+5e^{\beta}]^2} \rightarrow \ell(\beta) = ln\, L(\beta) = \beta - ln[5 + 5e^{\beta}] - 2\, ln[4 + 5e^{\beta}]$

$\frac{d}{d\beta}\, \ell(\beta) = 1 - \frac{5e^{\beta}}{5+5e^{\beta}} - 2\left(\frac{5e^{\beta}}{4+5e^{\beta}}\right) = 1 - \frac{5x}{5+5x} - 2\left(\frac{5x}{4+5x}\right)$, which we set equal to 0 in order to maximize $ln\, L(\beta)$. $1 - \frac{x}{1+x} - 2\left(\frac{5x}{4+5x}\right) = \frac{-10x^2-5x+4}{4+9x+5x^2} = 0 \rightarrow -10x^2 - 5x + 4 = 0$

$\rightarrow e^{\beta} = x = .43,\ -.93$. We discard the root $-.93$, since $e^{\beta} > 0$. Then the mle of β is $ln(.43) = -.8438$. Answer: C