EXAM P QUESTIONS OF THE WEEK

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Question 3 - Week of August 8

Six digits from 2 , 3 , 4 , 5 , 6 , 7 , 8 are chosen and arranged in a row without replacement to create a 6-digit number. Find the probabilities of the following events.

(a) The resulting number is divisible by 2.

(b) The digits 2 and 3 appear consecutively in order (i.e., 23 appears in the number).

(c) The digits 2 and 3 appear in order but not consecutively (i.e. 2 before 3, but at least one other number between them).

The solution can be found below.

Question 3 Solution

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Solution: To find the probability that a certain type of comination or arrangement occurs, the probability is usually formulated as $\frac{\text{number of combinations or arrangement of the specific type required}}{\text{total number of all combinations or arrangements}}$.
For these problems, the denominator is the total number of all 6 digit numbers that can be created by choosing 6 digits without replacement from 2 , 3 , 4 , 5 , 6 , 7 , 8. The total number of 6-digit numbers is $7 \times 6 \times 5 \times 4 \times 3 \times 2 = 5,040$ since the first digit can be any one of the 7 integers, the second digit can be any one of the remaining 6 integers, etc.

(a) The number is even if it ends in 2 , 4 , 6 or 8 . For each of these 4 cases, there are $6 \times 5 \times 4 \times 3 \times 2 = 720$ arrangements of the first 5 digits in the number, since the other 5 digits are chosen from the 6 remaining integers. The numerator of the probability is $4 \times 720 = 2880$, and the probability is $\frac{2880}{5040} = \frac{4}{7}$.
An alternative solution is to note that there are 7 possible equally likely final digits for the 6-digit number, and 4 of them make the number even. The probability is $\frac{4}{7}$.

(b) There are 5 available positions for the sequence 23 in the 6 digit number:
2 3 * * * * , * 2 3 * * * , * * 2 3 * * , * * * 2 3 * , * * * * 2 3
There are $5 \times 4 \times 3 \times 2 = 120$ ways of ordering the 4 integers in the * positions that are chosen from the remaining integers 4 , 5 , 6 , 7 , 8 .
The numerator of the probability is $5 \times 120 = 600$, and the probability is $\frac{600}{5040} = \frac{5}{42}$.

(c) There are $\binom{6}{2} = 15$ ordered positions for the 2 and 3 in the 6-digit number (2 3 * * * * , 2 * 3 * * * , . . . , * * * * 2 3), and 5 of them are consecutive (as in part (b) above).
Therefore, there are $\binom{6}{2} - 5 = 10$ ordered positions for 2 before 3 that are not consecutive.
As in (b), there are $5 \times 4 \times 3 \times 2 = 120$ ways of ordering 4 of the digits 4 , 5 , 6 , 7 , 8 .
The numerator is $10 \times 120 = 1200$, and the probability is $\frac{10 \times 120}{5040} = \frac{5}{21}$.