EXAM P QUESTIONS OF THE WEEK

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Question 6 - Week of August 29

X and Y are discrete random variables on the integers $\{0,1,2\}$, with moment generating functions $M_X(t)$ and $M_Y(t)$. You are given the following:
$M_X(t)+M_Y(t)=\frac{3}{4}+\frac{3}{4}e^t+\frac{1}{2}e^{2t}$ and $M_X(t)-M_Y(t)=\frac{1}{4}-\frac{1}{4}e^t$.
Find $P(X=1)$.
A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{3}{8}$ D) $\frac{1}{2}$ E) $\frac{5}{8}$

The solution can be found below.

Question 6 Solution

We denote the probability function of X by $p_0^X = P(X=0)$, $p_1^X = P(X=1)$, and $p_2^X = P(X=2)$. With similar notation for Y.

Then $M_X(t) = E[e^{tX}] = e^0 \cdot p_0^X + e^t \cdot p_1^X + e^{2t} \cdot p_2^X$ and

$M_Y(t) = E[e^{tY}] = e^0 \cdot p_0^Y + e^t \cdot p_1^Y + e^{2t} \cdot p_2^Y$.

Then $M_X(t) + M_Y(t) = p_0^X + e^t \cdot p_1^X + e^{2t} \cdot p_2^X + p_0^Y + e^t \cdot p_1^Y + e^{2t} \cdot p_2^Y$

$= p_0^X + p_0^Y + (p_1^X + p_1^Y)e^t + (p_2^X + p_2^Y)e^{2t}$,

and it follows that $p_0^X + p_0^Y = \frac{3}{4}$, $p_1^X + p_1^Y = \frac{3}{4}$ and $p_2^X + p_2^Y = \frac{1}{2}$.

In a similar way, we have

$M_X(t) - M_Y(t) = p_0^X + e^t \cdot p_1^X + e^{2t} \cdot p_2^X - p_0^Y - e^t \cdot p_1^Y - e^{2t} \cdot p_2^Y$

$= p_0^X - p_0^Y + (p_1^X - p_1^Y)e^t + (p_2^X - p_2^Y)e^{2t}$,

and it follows that $p_0^X - p_0^Y = \frac{1}{4}$, $p_1^X - p_1^Y = -\frac{1}{4}$ and $p_2^X - p_2^Y = 0$.

From these equations, we see that $p_1^X + p_1^Y + p_1^X - p_1^Y = 2\,p_1^X = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$,

and therefore $P(X=1) = p_1^X = \frac{1}{4}$. Answer: B