EXAM M QUESTIONS OF THE WEEK

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Question 6 - Week of August 29

Smith will be retiring at age 65 and is eligible to receive a pension annuity starting at age 65. The annuity will pay \$50,000 at the beginning of each year as long as Smith survives (first payment at age 65). Smith is married and his wife is 5 years younger than he is. Smith considers some alternative arrangements for his pension benefit.
You are given that $i = .04$, $\ddot{a}_{60} = 12$, $\ddot{a}_{65} = 10$, $\ddot{a}_{60:65} = 8$.
The following assumes that Smith and his wife are both alive when Smith turns 65 when his retirement benefit is to start. Equivalent benefits are based on actuarial equivalence.

(a) Smith considers a retirement annuity-due with payment of K per year as long as either he or his wife is alive. Find K.

(b) Smith considers a retirement annuity-due with payment of C per year while both he and his wife are alive, and with the payment reducing to $.75C$ after the first death, and continuing at that level until the second death. Find C.

(c) Smith considers the following combination of benefits:
(i) annuity-due of R per year while both he and his wife are alive, reducing to $\frac{1}{2}R$ after the first death, and continuing at that level until the second death, combined with
(ii) insurance payment of \$100,000 at the time of the first death.
Find R.

The solution can be found below.

Question 6 Solution

APV of original benefit is $50,000\ddot{a}_{65} = 500,000$

(a) $500,000 = K\ddot{a}_{\overline{60:65}} = K[\ddot{a}_{60} + \ddot{a}_{65} - \ddot{a}_{60:65}] = K[12 + 10 - 8] \rightarrow K = 35,714$.

(b) $500,000 = C\ddot{a}_{60:65} + .75C(\ddot{a}_{60} - \ddot{a}_{60:65}) + .75C(\ddot{a}_{65} - \ddot{a}_{60:65})$
$= 8C + .75C(12 - 8) + .75C(10 - 8) \rightarrow 12.5C = 500,000 \rightarrow C = 40,000$.

(c) $500,000 = 100,000A_{60:65} + R\ddot{a}_{60:65} + .5R(\ddot{a}_{60} - \ddot{a}_{60:65}) + .5R(\ddot{a}_{65} - \ddot{a}_{60:65})$
$A_{60:65} = 1 - d\ddot{a}_{60:65} = .69231$
$\rightarrow 500,000 = 69,231 + 8R + .5R(12 - 8) + .5R(10 - 8) \rightarrow R = 39,161$.