

EXAM M QUESTIONS OF THE WEEK

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A company insures the lives of two senior executives. The following assumptions are made.

- both executives have the same constant force of mortality
- mortality of the two executives is modeled with a common shock model
- the force of interest is 7.2%
- single benefit premium for \$1,000,000 continuous insurance on one of the lives is \$200,000
- single benefit premium for \$1,000,000 continuous insurance payable on the first death of the two lives is \$294,118

Find the expected time until the second death.

The solution can be found below.

Week of August 28/06 - Solution

μ_x^* , μ_y^* and λ are the common shock components, and we are given that $\mu_x^* = \mu_y^*$, which we will denote μ^* .

The overall force of mortality for (x) is $\mu^* + \lambda$ and same for (y) .

The force of mortality for the joint status of (x) and (y) is $\mu^* + \mu^* + \lambda = 2\mu^* + \lambda$.

The expected time until the second death is

$$\overset{\circ}{e}_{xy} = \overset{\circ}{e}_x + \overset{\circ}{e}_y - \overset{\circ}{e}_{xy} = \frac{1}{\mu^* + \lambda} + \frac{1}{\mu^* + \lambda} - \frac{1}{2\mu^* + \lambda}.$$

The single premium for a continuous whole life insurance of 1 for (x) is $\bar{A}_x = \frac{\mu^* + \lambda}{\mu^* + \lambda + .072}$ and the same for (y) . The single premium for a continuous whole life insurance of 1 for the joints status (xy) is $\bar{A}_{xy} = \frac{2\mu^* + \lambda}{2\mu^* + \lambda + .072}$.

We are given that based on common shock parameter λ , we have

$$\frac{\mu^* + \lambda}{\mu^* + \lambda + .072} = .200 \quad \text{and} \quad \frac{2\mu^* + \lambda}{2\mu^* + \lambda + .072} = .294118.$$

Writing these two equations as

$$\mu^* + \lambda = .2(\mu^* + \lambda + .072) \quad \text{and} \quad 2\mu^* + \lambda = .294118(2\mu^* + \lambda + .072)$$

gives us two equations in the two unknown quantities μ^* and λ .

Solving the equations results in $\mu^* = .012$ and $\lambda = .006$.

Expected time until second death is $\frac{1}{\mu^* + \lambda} + \frac{1}{\mu^* + \lambda} - \frac{1}{2\mu^* + \lambda} = 77.8$.