EXAM M QUESTIONS OF THE WEEK

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Question 5 - Week of August 22

1. A 4-year fully discrete term insurance with face amount 1000 is issued at age x (premiums are scheduled for the lifetime of the policy). The effective annual interest rate is $i = 25\%$, and the mortality probabilities are $q_x = .20$, $q_{x+1} = .25$, $q_{x+2} = .40$, $q_{x+3} = .50$.
The equivalence principle annual premium is $1000P^{1}_{x:\overline{4}|} = 219.45$.

(a) Formulate the 2nd year terminal prospective loss random variable (conditional distribution).

(b) Find the 2nd year terminal benefit reserve as the expected value of the loss random variable in part (a).

(c) Write the prospective form of the 2nd year terminal benefit reserve $1000\,{}_2V^{1}_{x:\overline{4}|}$, and calculate it by calculating each of the factors in the expression (the premium is given above).

(d) Write the retrospective form of the 2nd year terminal reserve , and calculate it by calculating each of the factors in the expression.

The solution can be found below.

Question 5 Solution

(a)

$$ {}_2L = \begin{cases} 1000v - P = 580.55 & \text{Prob. } q_{x+2} = .4 \\ 1000v^2 - P(1+v) = 245.00 & \text{Prob. } {}_{1|}q_{x+2} = (.6)(.5) = .3 \\ -P(1+v) = -395.00 & \text{Prob. } {}_2p_{x+2} = (.60(.5) = .3 \end{cases} $$

(b) ${}_2V = E[{}_2L] = (580.55)(.4) + (245.00)(.3) + (-395.00)(.3) = 187.22\,.$

(c) $1000\,{}_2V_{\overset{1}{x}:\overline{4}|} = 1000A_{\overset{1}{x+2}:\overline{2}|} - 1000P_{\overset{1}{x}:\overline{4}|} \cdot \ddot{a}_{x+2:\overline{2}|}$.

$A_{\overset{1}{x+2}:\overline{2}|} = vq_{x+2} + v^2\,{}_{1|}q_{x+2} = \frac{.4}{1.25} + \frac{(.6)(.5)}{(1.25)^2} = .512\,,\ \ddot{a}_{x+2:\overline{2}|} = 1 + vp_{x+2} = 1.48$.

$1000\,{}_2V_{\overset{1}{x}:\overline{4}|} = 1000(.512) - (219.45)(1.48) = 187.21$.

(d) $1000\,{}_2V_{\overset{1}{x}:\overline{4}|} = 1000P_{\overset{1}{x}:\overline{4}|} \cdot \ddot{s}_{x:\overline{2}|} - 1000 \cdot \frac{A_{\overset{1}{x}:\overline{2}|}}{v^2\,{}_2p_x}$.

$v^2\,{}_2p_x = \frac{(.8)(.75)}{(1.25)^2} = .384\ ,\ \ddot{s}_{x:\overline{2}|} = \frac{\ddot{a}_{x:\overline{2}|}}{v^2\,{}_2p_x} = \frac{1+vp_x}{v^2\,{}_2p_x} = \frac{1+\frac{.8}{1.25}}{.384} = 4.2708$,

$A_{\overset{1}{x}:\overline{2}|} = vq_x + v^2\,{}_{1|}q_x = \frac{.2}{1.25} + \frac{(.8)(.25)}{(1.25)^2} = .288$.

$1000\,{}_2V_{\overset{1}{x}:\overline{4}|} = (219.45)(4.2708) - 1000 \cdot \frac{.288}{.384} = 187.23$.