EXAM P QUESTIONS OF THE WEEK

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Question 2 - Week of August 1

For a particular disease, it is found that .1% of the population will develop the disease and 2% of the population has a family history of having the disease. A genetic test is devised to predict whether or not the individual will develop the disease. For those with a family history of the disease, 20% of the time the genetic test predicts that the individual will develop the disease and for those with no family history of the disease, 1% of the time the genetic test predicts that the individual will develop the disease. The genetic test is not perfect, and individuals are followed to determine whether or not they actually develop the disease. It is found that for those who have a family history of the disease and for whom the genetic test predicts the disease will develop, 80% actually develop the disease. It is also found that for those who have a family history of the disease and for whom the genetic test does not predict the disease will develop, 10% actually develop the disease. Find the probability that someone with a family history of the disease will develop the disease.

The solution can be found below.

Question 2 Solution

We denote events as follows:
F - an individual has a family history of the disease
T - the genetic test indicates that an individual will develop the disease
D - an individual will develop the disease
We are given the probabilities $P(D) = .001$ and $P(F) = .02$.
The language "for those with a family history of the disease, 20% of the time the genetic test predicts that the individual will develop the disease" describes the conditional probability
$P($the genetic test indicates that the individual will develop the disease$|$the individual has a family history of the disease$) = P(T|F) = .20$.
In a similar way $P(D|T \cap F) = .80$ and $P(D|T' \cap F) = .10$.
We are asked to find $P(D|F) = \frac{P(D \cap F)}{P(F)}$.

This can be formulated as
$P(D|F) = \frac{P(D \cap F)}{P(F)} = \frac{P(D \cap F \cap T) + P(D \cap F \cap T')}{P(F)} = = \frac{P(D \cap T \cap F) + P(D \cap T' \cap F)}{P(F)}$.

$$P(D \cap T \cap F) = P(D|T \cap F) \cdot P(T \cap F) = P(D|T \cap F) \cdot P(T|F) \cdot P(F)$$
$$= (.8)(.20)(.02) = .0032$$

and, since $P(T'|F) = 1 - P(T|F) = .80$,

$$P(D \cap T' \cap F) = P(D|T' \cap F) \cdot P(T' \cap F) = P(D|T' \cap F) \cdot P(T'|F) \cdot P(F)$$
$$= (.10)(.80)(.02) = .0016 \text{ .}$$

Then $P(D|F) = \frac{.0032 + .0016}{.02} = .24$.

Note that the information that $P(D) = .001$ and $P(T|F') = .01$ are not needed to solve the problem.