EXAM C QUESTIONS OF THE WEEK

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Question 2 - Week of August 1

You are given the following random sample of observations:
3 , 4 , 6 , 6 , 7 , 7 , 8 , 8 , 9 , 12

The data is a sample of 10 observations from a compound Poisson distribution for which the severity distribution is exponential. Apply the method of moments to estimate the Poisson parameter (mean) λ and the exponential parameter (mean) θ.

Question 2 Solution

Let S denote the compound Poisson random variable, and let N denote the Poisson frequency random variable with $E[N] = \lambda$, and let X denote the exponential severity random variable with $E[X] = \theta$.

Then $E[S] = E[N] \cdot E[X] = \lambda\theta$ and
$Var[S] = E[N] \cdot E[X^2] = E[N] \cdot Var[X] + Var[N] \cdot (E[X])^2$
$= \lambda(2\theta^2) = \lambda \cdot \theta^2 + \lambda \cdot \theta^2 = 2\lambda\theta^2$.

The first moment of the empirical distribution based on the given sample is
$\frac{3+4+6+6+7+7+8+8+9+12}{10} = 7.0$.
The variance of the empirical distribution is $\frac{1}{10}[(3-7)^2 + (4-7)^2 + \cdots + (12-7)^2] = 5.8$.

According to the method of moments, we have the moment equations
$E[S] = E[N] \cdot E[X] = \lambda\theta = 7.0$ and $Var[S] = E[N] \cdot E[X^2] = 2\lambda\theta^2 = 5.8$.
Dividing the second equation by the first results in $\frac{2\lambda\theta^2}{\lambda\theta} = 2\theta = \frac{5.8}{7.0}$, from which we get the estimate of θ, $\widehat{\theta} = .4143$. Then $\widehat{\lambda} = \frac{7.0}{\widehat{\theta}} = 16.9$.