

EXAM M QUESTIONS OF THE WEEK

S. Broverman, 2005

Question 4 - Week of August 15

Smith, who has just turned 62, takes a 3-year mortgage loan of \$100,000 on his home. The loan has an annual effective rate of interest of 6% and has scheduled level annual payments at the end of year for 3 years (the day before Smith's 63rd, 64th and 65th birthdays). The lender offers Smith mortgage insurance. The insurance policy will pay the mortgage balance owing at the end of the year of death if Smith dies before the final (3rd) mortgage payment is made. The insurance policy is based on an annual effective interest rate of 4% and the following life table.

$x :$	62	63	64	65
$\ell_x :$	1000	950	850	700

The insurance policy has a level annual premium payable at the start of the year. The premium is calculated so that the expected value of the issue date loss to the insurer is -1000 . Find the level annual premium for the insurance.

The solution can be found below.

Question 4 Solution

The annual loan payment is $\frac{100,000}{a_{\overline{3}|.06}} = 37,410.98$

The outstanding balances just before the payment is made at the end of years 1, 2 and 3 are
 $OB_1 = 37,410.98 \cdot \ddot{a}_{\overline{3}|.06} = 106,000.00$, $OB_2 = 37,410.98 \cdot \ddot{a}_{\overline{2}|.06} = 72,704.36$,
and $OB_3 = 37,410.98 \cdot \ddot{a}_{\overline{1}|.06} = 37,410.98$.

We make these calculations because these are the death benefits at the ends of years 1, 2 and 3. For instance, if death occurs during the first year, before the loan payments is made at the end of the year, then the amount owing at the end of the year (the death benefit) is the present value of the payment due at the end of the year along with the two future payments.

The APV of the death benefits is

$$106,000.00 v_{.04} q_{62} + 72,704.36 v_{.04}^2 {}_1|q_{62} + 37,410.98 v_{.04}^3 {}_2|q_{62} ,$$

$$\text{where } q_{62} = \frac{d_{62}}{\ell_{62}} = \frac{50}{1000} , {}_1|q_{62} = \frac{d_{63}}{\ell_{62}} = \frac{100}{1000} \text{ and } {}_2|q_{62} = \frac{d_{64}}{\ell_{62}} = \frac{150}{1000} .$$

The APV of the death benefits is 16,807 .

With annual premium of amount P , the APV of premiums is

$$P \ddot{a}_{\overline{62:\overline{3}}|} = P[1 + v_{.04} p_{62} + v_{.04}^2 {}_2p_{62}] = P[1 + \frac{.95}{1.04} + \frac{.85}{(1.04)^2}] = 2.6993P .$$

The expected issue date loss for the insurance policy is

$$APV \text{ benefit} - APV \text{ premium} = 16,807 - 2.6993P .$$

In order for this to be -1000 , we must have $P = 6,597$.