EXAM FM QUESTIONS OF THE WEEK

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Question 4 - Week of August 15

Smith is scheduled to retire on his 65th birthday. He was born January 1, 1946 and was hired by his present employer on January 1, 2001, with a starting salary of \$50,000 per year. His employer makes contributions to a pension fund on the first day of January, April, July and October until he retires. Each contribution is 1% of his annual salary. The pension fund earns a nominal annual rate of interest of 8% compounded quarterly. Smith's salary is scheduled to increase by 8% every January 1 starting in 2002. The final employer contribution is on October 1, 2010. Find the value of Smith's pension fund on his 65th birthday.

The solution can be found below.

Question 4 Solution

A general formula that we can use in solving this problem is the accumulated value of a geometrically increasing annuity. Suppose that an annuity has the following n annual payments: $1\ ,\ 1+r\ ,\ (1+r)^2\ ,\ \ldots,\ (1+r)^{n-1}$.
Suppose that these payments are accumulating in an account earning an interest rate of i per year. Then the accumulated value, at the time of the n-th payment is $\frac{(1+i)^n-(1+r)^n}{i-r}$
(this is valid if $i \neq r$; in the case that $i=r$, the accumulated value is $n(1+i)^{n-1}$).

In this problem, we have quarterly payments, but annual geometric increase in payment size. For each calendar year's payments, (Jan. 1, Apr. 1, Jul. 1, Oct. 1) we replace the four payments with a single equivalent payment on Dec. 31. In the first year (2000), the four quarterly payments are 500 each (1% of the salary of 50,000 in the first year). The quarterly rate of interest is 2%, so those four payments would accumulate to
$500[(1.02)^4+(1.02)^3+(1.02)^2+(1.02)] = 500\ddot{s}_{\overline{4}|.02} = 2,102.02$.
This would be the single payment on Dec. 31, 2001 that is equivalent to the 4 contributions made by the employer for 2001. The next year, 2002 can be dealt with in a similar way, except that all payments are 1.08 times as large as the previous year, so the equivalent single payment on Dec. 31, 2002 would be 1.08 times as large as 2,102.02 . In the 3rd year, 2003, the equivalent single payment on Dec. 31 would be 1.08 times as large again, so $2,102.02(1.08)^2$. We can continue in this way through the 10th year, 2010, and the equivalent single payment on Dec. 31,2010 would be $2,102.02(1.08)^9$.

We see that the employer contributions are equivalent to 10 annual contributions every Dec. 31 of $2,102.02\ ,\ 2,102.02(1.08)\ ,\ 2,102.02(1.08)^2\ ,\ \ldots\ ,\ 2,102.02(1.08)^9$.
The annual effective rate of interest being earned is $i=(1.02)^4-1=.082432$.
We can now use the geometric payment accumulation form mentioned above since the geometric period and the payment period coincide; the annual effective rate of interest is $i=.082432$, the geometric rate of increase is $r=.08$, and there are $n=10$ payments. The accumulated value of the 10 payments on Dec. 31, 2010 is $2,102.02 \cdot \frac{(1.082432)^{10}-(1.08)^{10}}{.082432-.08} = 42,448$.