

EXAM P QUESTIONS OF THE WEEK

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Week of April 3/06

In a one-car family consisting of a father, a mother and a son, the car usage is broken down in the following way: mother - 50% of the time, father - 40% of the time, son - 10% of the time.

An insurer models the possible loss that can occur as a result of an accident for this family in the following way. If a parent is the only driver during the year, there is a 1% chance of an accident occurring during the year. If the son is the only driver during the year, there is a 5% chance of an accident occurring during the year. The insurer will pay a claim on only one accident per year.

If there is an accident when the mother or father is driving, the amount of damage claim is uniformly distributed between 0 and 1, and if there is an accident when the son is driving, the pdf for the amount of damage claim is $f(x) = 2x$ for $0 < x < 1$.

Suppose that an accident occurs during the year and the amount of damage is at least .75. Find the probability that one of the parents was driving.

The solution can be found below.

Week of April 3/06 - Solution

Let X denote the amount of the damage claimed during the year.

Let P denote the event that a parent was driving at the time of the accident, and let S denote the event that the son was driving. The conditional pdf of damage given that a parent was driving and an accident occurred is $f(x|P \text{ and accident}) = 1$ for $0 < x < 1$, and the conditional pdf given that the son was driving and an accident occurred is $f(x|S \text{ and accident}) = 2x$ for $0 < x < 1$.

We wish to find the probability $P(P|X \geq .75) = \frac{P(P \cap X \geq .75)}{P(X \geq .75)}$.

The numerator is

$$\begin{aligned} P(P \cap X \geq .75) &= P(X \geq .75|P \text{ and accident}) \cdot P(P \text{ and accident}) \\ &= (.25) \cdot P(\text{accident}|P) \cdot P(P) = (.25)(.01)(.9) = .00225 . \end{aligned}$$

The denominator is

$$\begin{aligned} P(X \geq .75) &= P(P \cap X \geq .75) + P(S \cap X \geq .75) = .00225 + P(S \cap X \geq .75) , \text{ where} \\ P(S \cap X \geq .75) &= P(X \geq .75|S \text{ and accident}) \cdot P(S \text{ and accident}) \\ &= \left(\int_{.75}^1 2x \, dx\right) \cdot P(\text{accident}|S) \cdot P(S) = (.4375)(.05)(.1) = .0021875 . \end{aligned}$$

So the denominator is $.00225 + .0021875 = .0044375$.

$$\text{Then, } P(P|X \geq .75) = \frac{.00225}{.0044375} = .507 .$$