

EXAM FM QUESTIONS OF THE WEEK

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Week of April 3/06

Three 10-payment annuities-immediate are valued at annual effective rate of interest 10%,

I: level payments of 55 every year for 10 years,

II: increasing payments for 10 years with payment amounts 10, 20, 30, . . . , 90, 100,

III: decreasing payments for 10 years with payment amounts 100, 90, 80, . . . , 20, 10.

Rank the Macaulay duration of the three sets of cashflows from smallest to largest.

The solution can be found below.

Week of April 3/06 - Solution

The duration of a set of cashflows is the "weighted average time to maturity" of the set of cashflows. Larger payments at the start of the series of cashflows tends to make the duration lower and larger payments near the end of the series tends to make the duration larger. We note that the three series of cashflows all have the same total amount of 550 paid, so we would expect that the decreasing annuity has the smallest duration, the level annuity has the next largest duration and the increasing annuity would have the largest duration, $D_{III} < D_I < D_{II}$. The actual numerical of the Macaulay durations are

$$D_I = \frac{v+2v^2+3v^3+\dots+10v^{10}}{v+v^2+v^3+\dots+v^{10}} = \frac{(Ia)_{\overline{10}|}}{a_{\overline{10}|}} = 4.73$$

$$D_{II} = \frac{v+4v^2+9v^3+\dots+10v^{10}}{v+2v^2+3v^3+\dots+10v^{10}} = \frac{v+4v^2+9v^3+\dots+100v^{10}}{(Ia)_{\overline{10}|}} = 6.39$$

$$D_{III} = \frac{10v+18v^2+24v^3+\dots+10v^{10}}{10v+9v^2+8v^3+\dots+v^{10}} = \frac{10v+18v^2+24v^3+\dots+10v^{10}}{(Da)_{\overline{10}|}} = 3.47$$

The numerators of D_{II} and D_{III} can be found by direct calculation.