EXAM P QUESTIONS OF THE WEEK

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Week of April 24/06

X has a Poisson distribution with a mean of 2.
Y has a geometric distribution on the integers 0,1,2,..., also with mean 2.
X and Y are independent.
Find $P(X = Y)$.

The solution can be found below.

Week of April 24/06 - Solution

The probability function of X is $P(X=k)=\frac{e^{-2}\cdot 2^k}{k!}$.

The general probability function of a geometric distribution on $0,1,2,...$ is of the form $P(Y=k)=p(1-p)^k$ for $k=0,1,2,...$ and the mean is $\frac{1-p}{p}$.
Since the mean is 2, we have $\frac{1-p}{p}=2$, from which we get $p=\frac{1}{3}$,
so the probability function of Y is $P(Y=k)=(\frac{1}{3})(\frac{2}{3})^k$.

$$P(X=Y)=P(X=Y=0)+P(X=Y=1)+\cdots=\sum_{k=0}^{\infty}P(X=Y=k) \ .$$

Since X and Y are independent, we have
$P(X=Y=k)=P(X=k)\cdot P(Y=k)=\frac{e^{-2}\cdot 2^k}{k!}\cdot(\frac{1}{3})(\frac{2}{3})^k=\frac{e^{-2}}{3}\cdot\frac{(4/3)^k}{k!}$.

Then, $P(X=Y)=\sum_{k=0}^{\infty}P(X=Y=k)=\sum_{k=0}^{\infty}\frac{e^{-2}}{3}\cdot\frac{(4/3)^k}{k!}=\frac{e^{-2}}{3}\cdot\sum_{k=0}^{\infty}\frac{(4/3)^k}{k!}$.

The Taylor series expansion for e^x is $e^x=\sum_{k=0}^{\infty}\frac{x^k}{k!}$, so it follows that $\sum_{k=0}^{\infty}\frac{(4/3)^k}{k!}=e^{4/3}$.

Then, $P(X=Y)=\frac{e^{-2}}{3}\cdot e^{4/3}=\frac{e^{-2/3}}{3}$.