

EXAM C QUESTIONS OF THE WEEK

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Week of April 24/06

You are given the following table of data for three policyholders over a three year period.

| Policy Year → | | 1 | 2 | 3 |
|------------------------|--------------------|-----|-----|-----|
| Policyholder ↓ 1 | Number of Claims | 40 | 50 | |
| | Average Claim Size | 200 | 220 | |
| 2 | Number of Claims | 100 | 120 | 120 |
| | Average Claim Size | 200 | 200 | 150 |
| 3 | Number of Claims | 50 | 60 | |
| | Average Claim Size | 200 | 250 | |

Apply the nonparametric empirical Bayes credibility method to find the credibility premium per claim in the 4th year for Policyholder 2 using the standard method for μ (not the method that preserves total losses).

Solution can be found below.

Week of April 24/06 - Solution

$$\bar{X}_1 = \frac{(40)(200) + (50)(220)}{40 + 50} = 211.11, \quad m_1 = 90$$

$$\bar{X}_2 = \frac{(100)(200) + (120)(200) + (120)(150)}{100 + 120 + 120} = 182.35, \quad m_2 = 340$$

$$\bar{X}_3 = \frac{(50)(200) + (60)(250)}{50 + 60} = 227.27, \quad m_3 = 110$$

$$\hat{\mu} = \bar{X} = \frac{(40)(200) + (50)(220) + (100)(200) + (120)(200) + (120)(150) + (50)(200) + (60)(250)}{90 + 340 + 110} = 196.30$$

$$m = 540.$$

$$\hat{v}_1 = \frac{1}{2-1} \cdot [40(200 - 211.11)^2 + 50(220 - 211.11)^2] = 8888.89,$$

$$\hat{v}_2 = \frac{1}{3-1} \cdot [100(200 - 182.35)^2 + 120(200 - 182.35)^2 + 120(150 - 182.35)^2] = 97,058.82,$$

$$\hat{v}_3 = \frac{1}{2-1} \cdot [50(200 - 227.27)^2 + 60(250 - 227.27)^2] = 68,181.82$$

$$\hat{v} = \frac{\hat{v}_1 + 2\hat{v}_2 + \hat{v}_3}{1+2+1} = 67,797.1.$$

$$\hat{a} = \frac{1}{m - \frac{1}{m} \sum_{i=1}^r m_i^2} \cdot \left[\sum_{i=1}^r m_i (\bar{X}_i - \bar{X})^2 - \hat{v}(r-1) \right]$$

$$= \frac{1}{540 - \frac{1}{540}(90^2 + 340^2 + 110^2)}$$

$$\times [90(211.11 - 196.3)^2 + 340(182.35 - 196.3)^2 + 110(227.27 - 196.3)^2] - 67,797.1(2)$$

$$= 193.5.$$

The credibility premium for policyholder 2 is

$$\hat{Z}\bar{X}_2 + (1 - \hat{Z})\hat{\mu} = \left(\frac{340}{340 + \frac{67,797.1}{193.5}}\right) \cdot (182.35) + \left(1 - \frac{340}{340 + \frac{67,797.1}{193.5}}\right) (196.3) = 189.4.$$