EXAM P QUESTIONS OF THE WEEK

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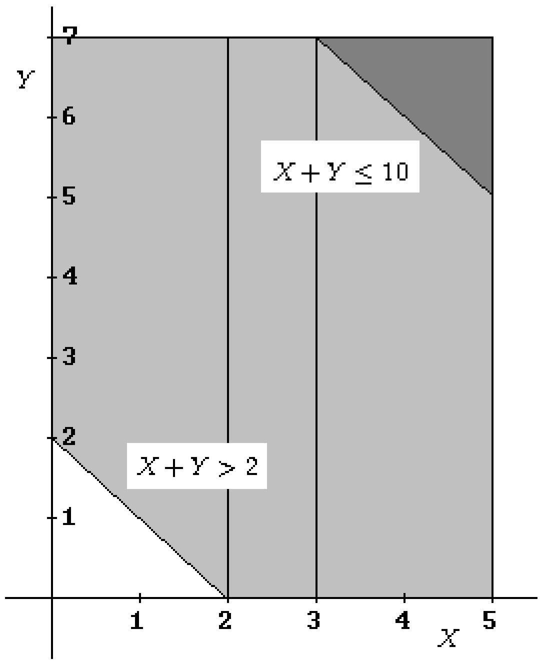
Week of April 17/06

A husband and wife have a health insurance policy. The insurer models annual losses for the husband separately from the wife. X is the annual loss for the husband and Y is the annual loss for the wife. X has a uniform distribution on the interval $(0,5)$ and Y has a uniform distribution on the interval $(0,7)$, and X and Y are independent. The insurer applies a deductible of 2 to the combined annual losses, and the insurer pays a maximum of 8 per year. Find the expected annual payment made by the insurer for this policy.

The solution can be found below.

Week of April 17/06 - Solution

The joint distribution of X and Y has pdf $f(x,y) = \frac{1}{5} \cdot \frac{1}{7} = \frac{1}{35}$ on the rectangle $0 < x < 5$ and $0 < y < 7$. The insurer pays $X + Y - 2$ if the combined loss $X + Y$ is > 2. The maximum payment of 8 is reached if $X + Y - 2 \geq 8$, or equivalently, if $X + Y \geq 10$. Therefore, the insurer pays $X + Y - 2$ if $2 < X + Y \leq 10$ (the lighter shaded region in the diagram below), and the insurer pays 8 if $X + Y > 10$ (the darker shaded region in the diagram below).



The expected amount paid by the insurer is a combination of two integrals:

$\int\int (x + y - 2) \cdot \frac{1}{35}\, dy\, dx$, where the integral is taken over the region $2 < x + y \leq 10$ (the lightly shaded region), plus

$\int\int 8 \cdot \frac{1}{35}\, dy\, dx$, where the integral is taken over the region $X + Y > 10$ (the darker region).

The second integral is $\frac{8}{35} \cdot (2) = \frac{16}{35}$, since the area of the darkly shaded triangle is 2 (it is a 2×2 right triangle).

The first integral can be broken into three integrals:

$$\int_0^2 \int_{2-x}^7 (x + y - 2) \cdot \frac{1}{35}\, dy\, dx + \int_2^3 \int_0^7 (x + y - 2) \cdot \frac{1}{35}\, dy\, dx + \int_3^5 \int_0^{10-x} (x + y - 2) \cdot \frac{1}{35}\, dy\, dx$$

$$= \frac{1}{35} \cdot \left[\int_0^2 \frac{(x+5)^2}{2}\, dx + \int_2^3 \frac{7(2x+3)}{2}\, dx + \int_3^5 \frac{60 + 4x - x^2}{2}\, dx \right]$$

$$= \frac{1}{35} \cdot \left[\frac{109}{3} + 28 + \frac{179}{3} \right] = \frac{124}{35}.$$

The total expected insurance payment is $\frac{16}{35} + \frac{124}{35} = \frac{140}{35} = 4$.