EXAM M QUESTIONS OF THE WEEK

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Week of April 17/06

An aggregate claims random variable S has a compound distribution for which the frequency has a geometric distribution, and the severity distribution is $X = \begin{cases} 1 & \text{prob. } q \\ 2 & \text{prob. } 1-q \end{cases}$.

The mean of S is 2.55 and the stop loss premium with a deductible of 1 is 1.95 .

Find the stop loss premium with a deductible of 2.

The solution can be found below.

Week of April 17/06 - Solution

Let us denote the geometric distribution parameter with the usual notation β.
Then $E[N] = \beta$ and $E[X] = 1 + 2(1-q) = 2 - q$,
and $E[S] = E[N] \times E[X] = \beta \times (2-q) = 2.55$.

$E[S \wedge 1] = 1 \times P(S \geq 1) = 1 - P(S=0) = 1 - P(N=0) = 1 - \frac{1}{1+\beta}$.

We are given that $E[(S-1)_+] = 1.95$, so that
$E[(S-1)_+] = E[S] - E[S \wedge 1] = \beta \times (2-q) - [1 - \frac{1}{1+\beta}] = 1.95$.

It follows that $1.55 + \frac{1}{1+\beta} = 1.95$ from which we get $\beta = 1.5$,
and then from $\beta \times (2-q) = 2.55$ we get $q = .3$.

The stop loss premium with a deductible of 2 is
$E[(S-2)_+] = E[S] - E[S \wedge 2] = 2.55 - [1 \times f_S(1) + 2 \times P(S \geq 2)]$.
$f_S(1) = P(S=1) = P(N=1) \times P(X=1) = \frac{\beta}{(1+\beta)^2} \times q = \frac{1.5}{(2.5)^2} \times (.3) = .072$,
and $P(X \geq 2) = 1 - f_S(0) - f_S(1) = 1 - .4 - .072 = .528$.
Then $E[(S-2)_+] = 2.55 - [.072 + 2(.528)] = 1.422$.