EXAM FM QUESTIONS OF THE WEEK

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Week of April 17/06

An actuary is analyzing price and duration of three separate bonds, Bond A, Bond B and Bond C, all based on a flat term structure (same yield to maturity for all bonds). Bond B has a price of 110 and a Macaulay duration of 11 years and Bond C has a price of 95 and a Macaulay duration of 14 years. The portfolio resulting from the combination of Bond A and Bond B has a Macaulay duration of 11.49 years, and the portfolio resulting from the combination of all three bonds has a Macaulay duration of 12.26 years. Find the price and Macaulay duration of Bond A.

The solution can be found below.

Week of April 17/06 - Solution

We will denote the bond prices and durations by P_A , P_B , P_C , D_A , D_B , and D_C .

Then $P_B = 110$, $D_B = 11$, $P_C = 95$ and $D_C = 14$.

For any series of cashflows, $D = \frac{X}{P}$, where $X = -v \times \frac{dP}{di}$,
so that $X = D \times P$.

Then $X_B = D_B \times P_B = (11)(110) = 1210$
and $X_C = D_C \times P_C = (14)(95) = 1330$.

For the portfolio made up of Bond A and Bond B,
$X_{A+B} = D_{A+B} \times P_{A+B} = (11.49)(P_A + 110) = 1263.9 + 11.49P_A$.
But
$X_{A+B} = -v \times \frac{dP_{A+B}}{di} = -v \times \frac{d(P_A+P_B)}{di} = -v \times (\frac{dP_A}{di} + \frac{dP_B}{di}) = X_A + X_B$,
Therefore, $X_{A+B} = D_A \times P_A + D_B \times P_B$
and $D_{A+B} = \frac{X_{A+B}}{P_{A+B}} = \frac{D_A \times P_A + D_B \times P_B}{P_A + P_B} \rightarrow 11.49 = \frac{D_A \times P_A + 1210}{P_A + 110}$.
This equation can be written as $D_A \times P_A - 11.49P_A = 53.9$.

In a similar way, we get
$D_{A+B+C} = \frac{X_{A+B+C}}{P_{A+B+C}} = \frac{D_A \times P_A + D_B \times P_B + D_C \times P_C}{P_A + P_B + P_C} \rightarrow 12.26 = \frac{D_A \times P_A + 1210 + 1330}{P_A + 110 + 95}$,
from which we get $D_A \times P_A - 12.26P_A = -26.7$.

From the two equations $D_A \times P_A - 11.49P_A = 53.9$ and $D_A \times P_A - 12.26P_A = -26.7$,
we get $P_A = 104.68$, and $D_A = 12.0$.