EXAM M QUESTIONS OF THE WEEK

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Week of April 3/06

Aggregate claims S follow a compound distribution with a Poisson frequency distribution N with mean 1, and a geometric severity distribution X with probability function
$P(X=j)=\frac{\beta^j}{(1+\beta)^{1+j}}$, for $j=0,1,2,...$
It is found that $E[S|S>0]=\frac{1}{2-2e^{-1/3}}$.
Determine $E(X)$.

The solution can be found below.

Week of April 3/06 - Solution

$E(X) = \beta$.

S is a non-negative integer-valued random variable with probability function

$P(S = k) = g_k$.

$E[S|S > 0] = \sum_{k=1}^{\infty} k \cdot P(S = k|S > 0) = \sum_{k=1}^{\infty} k \cdot \frac{P(S=k)}{1-P(S=0)} = \frac{E(S)}{1-P(S=0)}$.

The severity distribution is geometric with $P(X = 0) = \frac{1}{1+\beta}$.

$E(S) = E(N) \cdot E(X) = (1)(\beta) = \beta$.

$P(S = 0) = g_0 = \sum_{k=0}^{\infty} P(S = 0|N = k) \cdot P(N = k)$

$= \sum_{k=0}^{\infty} P(X_1 = 0 \cap X_2 = 0 \cap \cdots \cap X_k = 0|N = k) \cdot P(N = k) = \sum_{k=0}^{\infty} \frac{(\frac{1}{1+\beta})^k e^{-1} (1)^k}{k!}$

$= e^{-1} \cdot e^{1/(1+\beta)} = e^{-\beta/(1+\beta)}$.

$E[S|S > 0] = \frac{E(S)}{1-P(S=0)} = \frac{\beta}{1-e^{-\beta/(1+\beta)}} = \frac{1}{2-2e^{-1/3}} = \frac{.5}{1-e^{-.5/(1+.5)}}$.

We see that $\beta = .5$.