NOVEMBER 2005 EXAM FM SOA/CAS 2 SOLUTIONS

1. The simple interest rate of 8% suggests that it is the dollar-weighted rate of return. The dollar weighted equation is $25(1.08) + (X - 2.95)(1.04) = 25 + (X - 2.95) + 2$ (in millions). The right side of the equation is the total net amount deposited during the year plus the investment income for the year. Solving for X results in $X = 2.95$. Therefore, the only amount on deposit for the year is the initial 25 million since the sales revenue is cancelled by the salaries and other expenses at mid-year. The 25 million on deposit for the year grows to 27 million at the end of the year for an annual effective yield rate of 8%. Answer: D

2. The bond price is (PV) is $10v + 10v^2 + \cdots 10v^8 + 100v^8$.
Macaulay duration is $\frac{10v+20v^2+\cdots+80v^8+800v^8}{10v+10v^2+\cdots+10v^8+100v^8} = \frac{10(Ia)_{\overline{8}|}+800v^8}{10a_{\overline{8}|}+100v^8}$.
$a_{\overline{8}|.08} = 5.747$, $(Ia)_{\overline{8}|.08} = \frac{\ddot{a}_{\overline{8}|}-8v^8}{i} = 23.553$, so the duration is $\frac{667.74}{111.49} = 6$.
Answer: C

3. The accumulated value of at the end of 2 years (24 months) is $50\ddot{s}_{\overline{24}|j}$,
The AV at the end of 4 years is $50\ddot{s}_{\overline{24}|j}(1+i)^2 + 100\ddot{s}_{\overline{24}|j}$ and at the end of 6 years, it is
$50\ddot{s}_{\overline{24}|j}(1+i)^4 + 100\ddot{s}_{\overline{24}|j}(1+i)^2 + 150\ddot{s}_{\overline{24}|j} = 50\ddot{s}_{\overline{24}|j}[(1+i)^4 + 2(1+i)^2 + 3]$.
The AV at the end of 7 years is $1+i$ times as large as that. Answer: C

4. $118.20 = Cv^{20}_{.03} + 4a_{\overline{20}|.03} \rightarrow C = 106.00$.
Note that we can solve this using the N, PV , I and PMT functions to compute FV.
Answer: D

5. Under certain assumptions regarding the behavior of a stock's price, a riskless hedge can be created by combining the sale of certain number of call options with the purchase of a share of stock. The combined value of the portfolio after a change in stock price will be the same no matter what value the stock price changes to. This topic is covered in Section 8.2.3 of the book "Mathematic of Investment and Credit" by S. Broverman (hey, that's me!), and is also covered in Section 10.6 of "The Theory of Interest" by S. Kellison (the other guy). Answer: E

6. An investment of 1 would accumulate to $(1+i_4)^4 = (1.082)^4 = 1.370595$ at the end of 4 years. An investment of 1 would accumulate to $(1+i_5)^5 = (1.075)^5 = 1.435629$ at the end of 5 years. The implied one-year effective rate for the 5th year is j, where $1.370595(1+j) = 1.435629$, so that $j = .0474$. Answer: A

7. The annual effective rate for the 1 year certificate is $(1.01)^4 - 1 = .0406$, for the 3-year certificate it is $(1.0125)^4 - 1 = .0509$, and for the 5-year certificate it is $(1.014125)^4 - 1 = .0577$. In order to withdraw the investment at the end of 6 years, the investor must choose one of the following patterns of investment:
(i) 6 successive one-year certificates , annual effective rate is .0406 .
(ii) a 3 year certificate combined with 3 one-year certificates (in any order), annual effective rate is $[(1.0125)^{12}(1.01)^{12}]^{1/6} - 1 = .0458$ (we have found the 6-year accumulation and then the equivalent annual effective rate that would compound to the same amount in 6 years).
(iii) Two 3-year certificates, annual effective rate .0509 .
(iv) A one-year certificate and a 5-year certificate (either order), annual effective rate is $[(1.014125)^{20}(1.01)^4]^{1/6} - 1 = .0548$.
The maximum annual effective return is .0548 and is obtained with a 5-year and a 1-year certificate, in either order. Answer: D

8. The payments are

Time	0	1	$\cdots$	9	10	$\cdots$	19
	100	$100(1.05)$	$\cdots$	$100(1.05)^9$	$100(1.05)^9(.95)$	$\cdots$	$100(1.05)^9(.95)^{10}$

If we did the valuation one period before time 0, and if we value only the first 10 payments, then the pv of the first 10 payments is $100 \cdot \frac{1-(\frac{1.05}{1.07})^{10}}{.07-.05} = 859.76$. The value at time 0 of the first 10 payments is $859.76(1.07) = 919.94$.
The pv at time 9 of the final 10 payments is $100(1.05)^9(.95) \cdot \frac{1-(\frac{.95}{1.07})^{10}}{.07-(-.05)} = 854.33$.
The pv at time 0 (9 yeas earlier) of the second 10 payments is $854.33v_{.07}^9 = 464.70$.
The total pv at time 0 is $919.94 + 464.70 = 1384.64$. Answer: B

9. The yield rate i (annual effective) is the rate that makes the pv of the deposits equal to the pv of the payments received. The pv of the deposits is $1000+150\cdot\frac{1}{i}$.
The payments received form an geometrically increasing perpetuity-immediate, with payment growth rate 5%. The pv of the payments received is $100\cdot\frac{1}{i-.05}$.
Setting the two pv's equal results in $1000+150\cdot\frac{1}{i}=100\cdot\frac{1}{i-.05}$.
This becomes the equation $1000i^2-7.5=0$, from which we get $i=.0866$.
Note that once we have the equation $1000+150\cdot\frac{1}{i}=100\cdot\frac{1}{i-.05}$, we can substitute in the possible answers to see which one satisfies the equation. Answer: E

10. To match liabilities, the company will buy a 1-year zero coupon bond with face amount 1000, and a 2-year zero-coupon bond with face amount 2000. The cost (pv) of the purchase is $\frac{1000}{1.1}+\frac{2000}{(1.12)^2}=2503$. Answer: C

11. The price of the bond is $1000v_{.03}^{20}+40a_{\overline{20}|.03}=1148.77$. This is the amount borrowed, so the amount repaid at the end of 10 years is $1148.77(1.05)^{10}=1871.23$.
The reinvested coupons grow to $40s_{\overline{20}|.02}=971.89$, so that when the redemption amount of 1000 is received at the end of 10 years (the maturity date of the bond) and the loan is repaid, the net amount the investor has is $971.89+1000-1871.23=100.66$. Answer: B

12. The interest rate on Megan's perpetuity is i, where $3250=\frac{130}{i}$, so that $i=.04$.
The sequence of payments for Chris's annuity is

Time 0	1	2	$\cdots$	19	20
	P	$P+15$	$\cdots$	$P+270$	$P+2285$

We can write this sequence as a combination of two sequences of payments

	$P-15$	$P-15$	$\cdots$	$P-15$	$P-15$
	15	30	$\cdots$	285	300

The pv is $(P-15)a_{\overline{20}|.04}+15(Ia)_{\overline{20}|.04}=3250$.
Solving for P results in $P=116$. Answer: B

13. Let j denote the 3-month interest rate. Then $10,000 = 400a_{\overline{40}|j}$.
Using the calculate interest function, we get $j = .02524$.
The equivalent annual effective rate of interest will be $(1.02524)^4 - 1 = .1048$.
The equivalent monthly rate of interest is k, where $(1+k)^{12} = 1.1048$, so that
$k = .00834$, and the nominal annual interest rate convertible monthly is
$12 \times .00834 = .100$. Answer: A

14. Each time interest is generated from the primary account it is put into the secondary account earning 6%, but the principal deposits of X each remain in the primary account. The amounts in the primary account are X after the first deposit
The original payments and the interest generated is illustrated in the following time diagram.

	0	1		2		3	$\ldots$	19	20
Deposit	X	X	X		X		$\ldots$	X	
Interest			$.08X$		$2(.08X)$	$3(.08X)$		$\ldots$	$20(.08X)$

The interest payments are reinvested at 6%. The accumulated value of the reinvested interest is $(.08X)(Is)_{\overline{20}|.06} = 25.3236X$. The total accumulated value at the end of 20 years is
$20X + 25.3236X = 5600$, so that $X = 123.56$. Answer: B

15. We wish to find the pv at time 1 of payments of 5000 each at times 2, 3 and 4.
The implied accumulation from time 1 to time 2 is $1+f$, where $(1.05)(1+f) = (1.0575)^2$;
the right side is the accumulation for two years at the two-year spot rate, and the left side is accumulation for the first year at the one-year spot rate followed by the forward rate from time 1 to time 2. Therefore, the pv factor from time 2 back to time 1 is $(1+f)^{-1} = \frac{1.05}{(1.0575)^2}$.
The implied accumulation from time 1 to time 3 is $1+g$, where $(1.05)(1+g) = (1.0625)^3$;
the right side is the accumulation for three years at the three-year spot rate, and the left side is accumulation for the first year at the one-year spot rate followed by the forward growth from time 1 to time 3. Therefore, the pv factor from time 3 back to time 1 is $(1+g)^{-1} = \frac{1.05}{(1.0625)^3}$.
The implied accumulation from time 1 to time 4 is $1+h$, where $(1.05)(1+h) = (1.0650)^4$.
Therefore, the pv factor from time 3 back to time 1 is $(1+g)^{-1} = \frac{1.05}{(1.0650)^4}$.
The pv at time 1 of the annuity is $5000[\frac{1.05}{(1.0575)^2} + \frac{1.05}{(1.0625)^3} + \frac{1.05}{(1.0650)^4}] = 13,152.50$.
Note that we found 1, 2 and 3-year forward growth (and pv) factors from time 1. Answer: B

16. At the end of 10 years, Dan has the redemption amount of 1000 plus the reinvested coupons. The reinvested coupons accumulate to $45s_{\overline{20}|.035} = 1,272.59$, so Dan has 2,272.59 at the end of 10 years. Suppose his 6-month yield rate for the 10-year period is j. Then $925(1+j)^{20} = 2,272.59$, so that $j = .0460$, and the nominal annual yield convertible semiannually is $2 \times .046 = .092$. Note that this form of yield is the compound return rate that Dan realizes on his 925 initial investment taking into account the reinvestment of coupons. It is not the same as the yield rate that is used to value the bond initially. Answer: C

17. Theo deposits $25,000 \times .4 = 10,000$ into the margin account. The amount Theo has a year later after the short sale is completed is $10,000(1.08) + 25,000 - X$. Since Theo initially invested 10,000, and since we are told that Theo earned 25% on the transaction, Theo must have 12,500 after the short sale is completed. Therefore, $10,000(1.08) + 25,000 - X = 12,500$. Solving for X results in $X = 23,300$. Answer: D

18. The level annual payment is $789 + 211 = 1000$. Since this is a level payment loan, the principal repaid grows by 1.07 with every successive payment. Therefore, the principal in the 18th payment is $211(1.07)^{10} = 415.07$, so the interest in the 18th payment is $1000 - 415.07 = 585.93$. Answer: D

19. I and II are true, III is false. Answer: E

20. The pv (at time 0) of the first 5 dividends is $a_{\overline{5}|.06} = 4.21236$.
The pv (at time 0) of the next 5 dividends is $2v^5 \cdot a_{\overline{5}|.06} = 6.295$.
The 11th, 12th, 13th, etc dividends are $2(1.02)$, $2(1.02)^2$, $2(1.02)^3, \ldots$
The pv at time 10 of the 11th, 12th, 13th, . . . dividends is $\frac{2(1.02)}{.06-.02} = 51$.
The pv at time 0 of the 11th, 12th, 13th, . . . is $51 \cdot v^{10} = 28.478$.
The total pv at time 0 is $4.21 + 6.30 + 28.48 = 39$. Answer: D

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21. I is false. The convexity of the assets must be greater than the convexity of the liabilities.
II is true. This is definition of full immunization.
III. True.
Answer: D

22. Let n denote the number of 6-month periods until the bond is called. Then
$918 = 1100v_{.05}^{n} + 45a_{\overline{n}|.05}$. Using the calculator function,we get $n = 49.35$ six month periods, or 24.7 years. Answer: B

23. The present value is $1000(Da)_{\overline{25}|.1} = 1000 \cdot \frac{25 - a_{\overline{25}|.1}}{.1} = 15,923$. Answer: C

24. Let the 3-month yield rate be j. The equation of value is $850 = 1000v_j^{120} + 30a_{\overline{120}|j}$.
Using the calculator interest function, we get $j = .0354$.
The nominal annual yield rate is $4 \times .0354 = .142$. Answer: C

25. For the 1-year old, the amount needed now is $Xv^{17} + Yv^{20}$.
For the 3-year old, the amount needed now is $Xv^{15} + Yv^{18}$.
For the 6-year old, the amount needed now is $Xv^{12} + Yv^{15}$.
The total amount needed now is $X[v^{17} + v^{15} + v^{12}] + Y[v^{20} + v^{18} + v^{15}]$. Answer: E

