# MAY 2007 SOA EXAM MLC SOLUTIONS

1. ${}_3p_{70} = p_{70} \cdot {}_2p_{71} \to p_{70} = \frac{.95}{.96}$ .
${}_4p_{71} = e^{-\int_{71}^{75} \mu_x \, dx} = e^{-.107}$ .
${}_5p_{70} = p_{70} \cdot {}_4p_{71} = \frac{.95}{.96} \times e^{-.107} = .889$ .    Answer: E

2. $Var(\overline{a}_{\overline{T(x)|}}) = \frac{1}{\delta^2} \cdot ({}^2\overline{A}_x - \overline{A}_x^2)$
Since the force of mortality is constant at $c$, we have $\overline{A}_x = \frac{c}{\delta+c}$ and ${}^2\overline{A}_x = \frac{c}{2\delta+c}$.
Therefore, from $\overline{A}_x = .3443 = \frac{c}{.08+c}$ , we get $c = .042$ ,
and then ${}^2\overline{A}_x = \frac{.042}{2(.08)+.042} = .2079$ .
$Var(\overline{a}_{\overline{T(x)|}}) = \frac{1}{(.08)^2} \cdot [.2079 - (.3443)^2] = 13.96$ .    Answer: B

3. ${}_5V_{[60]} = A_{65} - P_{[60]} \cdot \ddot{a}_{65}$ (since we are past the select period of 3 years, the insurance annuity reverts to ultimate values). We can find $P_{[60]}$ from
$P_{[60]} = \frac{A_{[60]}}{\ddot{a}_{[60]}} = \frac{dA_{[60]}}{1-A_{[60]}} = \frac{\frac{.06}{1.06}(.359)}{1-.359} = .0317$ .
From the Illustrative Table we have $A_{65} = .43980$ and $\ddot{a}_{65} = 9.8969$ , so that the reserve for face amount 1 is ${}_5V_{[60]} = .43980 - (.0317)(9.8969) = .1261$ .
Multiplying by 1000 gives the reserve for the face amount 1000.    Answer: D

4. Since this is a fully discrete whole life insurance, for face amount 1, the variance of $L$ is $(\frac{1}{1-A_x})^2({}^2A_x - A_x^2) = .011487$ , and the standard deviation is $\sqrt{.011487} = .107177$ .
For face amount 150,000, the standard deviation is scaled up by a factor of $150,000$ to $150,000(.107177) = 16,077$ .    Answer: E

5. The exponential interarrival times with mean time $\mu$ between arrivals is equivalent to arrivals following a Poisson process with a mean of $\frac{1}{\mu}$ per unit time. We are given that the average interarrival time is 1 month, so the average number of arrivals per month is 1. Because of the independence of arrivals in disjoint intervals of time, the fact that there have been no arrivals by the end of January has no effect on how many arrivals will occur in February and March. The number of arrivals in Feb. and Mar. is Poisson with a mean of 2. The probability of at least 3 arrivals in Feb. and Mar. is the complement of the probability of at most 2 arrivals. This is $1 - (e^{-2} + \frac{e^{-2}2}{1!} + \frac{e^{-2}2^2}{2!}) = .323$ .    Answer: C

 

6. The units donated and the units withdrawn are independent of one another. The units donated follows a compound Poisson process and so do the units withdrawn. The mean of a compound Poisson distribution is $E[N] \cdot E[X]$ and the variance is $E[N] \cdot E[X^2]$ , where $E[N]$ is the Poisson mean, and $X$ is the amount of an individual deposit (or withdrawal for the withdrawal process). For the deposits in one week, $N_D$ has a mean of $7(10)(.8) = 56$ (since 80% of food bank visitors make a deposit) and $X_D$ has mean 15 and variance 75. For the withdrawals in one week, $N_W$ has a mean of $7(10)(.2) = 14$ (since 20% of food bank visitors make a withdrawal) and $X_D$ has mean 40 and variance 533. The expected amount deposited in one week is
$E[S_D] = E[N_D] \cdot E[X_D] = (56)(15) = 840$ and the variance of the amount deposited is
$Var[S_D] = E[N_D] \cdot E[X_D^2] = (56)(75 + 15^2) = 16,800$
(since $E[X_D^2] = VarpX_D] + E[X_D]^2$).
Similarly, the expected amount withdrawn in one week is
$E[S_W] = E[N_W] \cdot E[X_W] = (14)(40) = 560$ and the variance of the amount withdrawn is
$Var[S_W] = E[N_W] \cdot E[X_W^2] = (14)(533 + 40^2) = 29,862.$
The net amount deposit in the week is $S_D - S_W$, which has a mean of
$840 - 560 = 280$ and a variance of $16,800 + 29,863 = 46,662$ (because of independence of $S_D$ and $S_W$). The probability that the amount of food units at the end of 7 days will be at least 600 more than at the beginning of the week is $P(S_D - S_W \geq 600)$ .
Using the normal approximation, this
$P(\frac{S_D - S_W - 280}{\sqrt{46,662}} \geq \frac{600-280}{\sqrt{46,662}}) = 1 - \Phi(\frac{600-280}{\sqrt{46,662}}) = 1 - \Phi(1.48) = 1 - .9306 = .0694$ .
Answer: A

7. The earlier premium is paid, the higher the reserve will be. This can be seen retrospectively, since the accumulated cost of insurance is the same in all cases (level benefit of 1000), so the reserves differ because of different premium payment patterns. Earlier premium payment results in greater accumulation to time 5. Pattern E has the most premium paid earliest. E has the same total in the first 3 years as A and C and the same premium in years 4 and 5, so E's accumulated premium will be greater than that of A and C. The difference between E and D is that E has premium of 1 more than D in the first year and 1 less than D in the 3rd year, but D has one more than E in the 4th year and 1 less than E in the fifth year. Since E's excess differential with D occurs earlier (years 1 and 3, vs years 4 and 5), the accumulation of E's premium is greater than that of D. From the diagram, it can be seen that D's accumulated premium is greater than that of B. Answer: E

 

8. The expected number of points that Kira will score is
Prob. that Kira gets to play $\times$ Expected number of points Kira scores given that she starts to play
If Kira gets to play, the expected time until she will be called is $e_x^{Kira} = \frac{1}{\mu_{Kira}} = \frac{1}{.6} = \frac{5}{3}$ hours.
The expected number of points she would score in that time is $100,000(\frac{5}{3}) = 166,667$ .
The probability that Kira will get to play is the probability that Kevin gets called first. This is
$_\infty q_{\overset{1}{x}y}$ , where $x$ is Kevin and $y$ is Kira. This probability is
$_\infty q_{\overset{1}{x}y} = \int_0^\infty {}_tp_x \mu_x(t) \, {}_tp_y \, dt = \int_0^\infty e^{-.7t}(.7)e^{-.6t} \, dt = \frac{.7}{.13} = .538462$ .
The expected number of points Kira will score before she leaves is
$(.538462)(166,667) = 89,744$ . Answer: E

9. We first find $q_{25}^{(1)}$, the decrement probability for the continuous decrement.
$q_{25}^{(1)} = \int_0^1 {}_tp_{25}^{(\tau)} \cdot \mu_{25}^{(1)}(t) \, dt = \int_0^1 {}_tp_{25}^{\prime(1)} \cdot {}_tp_{25}^{\prime(2)} \cdot \mu_{25}^{(1)}(t) \, dt = q_{25}^{\prime(1)} \int_0^1 {}_tp_{25}^{\prime(2)} \, dt$ .
The last inequality follows from UDD in associated single tables for decrement 1.
${}_tp_{25}^{\prime(2)} = 1$ for $0 \leq t < \frac{1}{5}$ , since decrement 2 does not occur until time $\frac{1}{5}$.
Then ${}_tp_{25}^{\prime(2)} = 1 - (.12)(\frac{3}{4}) = .91$ for $\frac{1}{5} \leq t < \frac{3}{5}$ ,because $\frac{1}{4}$ of decrement 2 occurs at time $\frac{1}{5}$ and no more of decrement 2 occurs until time $\frac{3}{5}$.
Then ${}_tp_{25}^{\prime(2)} = 1 - (.12) = .88$ for $\frac{3}{5} \leq t \leq 1$ , because the rest of decrement 2 occurs at time $\frac{3}{5}$.
Then $q_{25}^{(1)} = q_{25}^{\prime(1)} \int_0^1 {}_tp_{25}^{\prime(2)} \, dt = (.1)[(1)(\frac{1}{5}) + (.91)(\frac{2}{5}) + (.88)(\frac{2}{5})] = .0916$ .
We know that for a 2-decrement model
$q_{25}^{(\tau)} = q_{25}^{\prime(1)} + q_{25}^{\prime(2)} - q_{25}^{\prime(1)} \cdot q_{25}^{\prime(2)} = .1 + .12 - (.1)(.12) = .208$ ,
and we also know that $q_{25}^{(\tau)} = q_{25}^{(1)} + q_{25}^{(2)}$ , so that $.208 = .0916 + q_{25}^{(2)}$ ,
from which we get $q_{25}^{(2)} = .1164$ . Answer: E

10. We are given that $300 = 1000A_{66}^{08}$ , where "08" refers to valuation at the start of 2008.
We wish to find $1000A_{65}^{08}$ . Using the recursive insurance relationship,
$A_{65}^{08} = v_{.1} \, q_{65} + v_{.1} \, p_{65} A_{66}^{09}$ , so we need to find $A_{66}^{09}$.
Again using the recursive relationship, we have $A_{66}^{09} = v_{.06} \, q_{66} + v_{.06} \, p_{66} A_{67}^{10}$ ,
so if we can find $A_{67}^{10}$ then we can get $A_{66}^{09}$, and then get $A_{65}^{08}$.
Applying the recursive relationship to $A_{66}^{08}$, we get
$A_{66}^{08} = v_{.1} \, q_{66} + v_{.1} \, q_{66} A_{67}^{09}$ , so that $.3 = \frac{1}{1.1}(.012) + \frac{1}{1.1}(.988) A_{67}^{09}$ ,

10 continued

and we get $A_{67}^{09} = .321862$ . Since the interest remains at 6% for 2009 and thereafter, it follows that $A_{67}^{09}$ is the same as $A_{67}^{10}$, so $A_{67}^{10} = .321862$ .

Then $A_{66}^{09} = v_{.06}\, q_{66} + v_{.06}\, p_{66} A_{67}^{10} = \frac{.012}{1.06} + \frac{.988}{1.06}(.321862) = .3113$ .

Finally, $A_{65}^{08} = v_{.1}\, q_{65} + v_{.1}\, p_{65} A_{66}^{09} = \frac{.01}{1.1} + \frac{.99}{1.1}(.3113) = .2893$ . Answer: C

11. $G\ddot{a}_{40:\overline{10}|} = 1000 A^{\;1}_{40:\overline{20}|} + .29G + .09G a_{40:\overline{9}|} + 10 + 5a_{40:\overline{19}|}$ .

Using the relationship $\ddot{a}_{40} = \ddot{a}_{40:\overline{10}|} + {}_{10}E_{40} \cdot \ddot{a}_{50}$ , from the Illustrative Table, we get

$14.6864 = \ddot{a}_{40:\overline{10}|} + (.53667)(13.2668)$ , so that $\ddot{a}_{40:\overline{10}|} = 7.5665$ .

Using the relationship $\ddot{a}_{40} = \ddot{a}_{40:\overline{20}|} + {}_{20}E_{40} \cdot \ddot{a}_{60}$ , from the Illustrative Table, we get

$14.6864 = \ddot{a}_{40:\overline{20}|} + (.27414)(11.1454)$ , so that $\ddot{a}_{40:\overline{20}|} = 11.3610$ .

We also use $A_{40} = A^{\;1}_{40:\overline{20}|} + {}_{20}E_{40} \cdot A_{60}$ , so from the Illustrative Table we get

$.16132 = A^{\;1}_{40:\overline{20}|} + (.27414)(.36913)$ , so that $A^{\;1}_{40:\overline{20}|} = .06013$ .

Then, $a_{40:\overline{9}|} = \ddot{a}_{40:\overline{10}|} - 1 = 6.5665$ and $a_{40:\overline{19}|} = \ddot{a}_{40:\overline{20}|} - 1 = 10.3610$ .

Substituting these values into the original equation results in

$7.5665G = 60.13 + .29G + .09G(6.5665) + 10 + 5(10.361)$ , and solving for $G$ results in

$G = 18.24$ . Answer: A

12. Given that $K(55) \geq 1$ (means that (55) is still alive at age 56) ${}_1L$ is a 5-point random variable as of age 56. The 5 possible values for ${}_1L$ are

$$
{}_1L = \begin{cases} 2000v - 50 = 1836 & \text{if death is acc. at age 56, prob. } q_{56}^{(1)} = .005 \\ 1000v - 50 = 893 & \text{if death is not acc. at age 56, prob. } q_{56}^{(1)} = .04 \\ 2000v^2 - 50(1+v) = 1683 & \text{if death is acc. at age 57, prob. } {}_{1|}q_{56}^{(1)} = (.955)(.008) \\ 1000v^2 - 50(1+v) = 793 & \text{if death is not acc. at age 57, prob. } {}_{1|}q_{56}^{(2)} = (.955)(.06) \\ -50(1+v) = -97 & \text{if (56) survives to age 58, prob. } {}_2p_{56}^{(\tau)} = (.955)(.932) \end{cases}
$$

From this table, we see that $P[{}_1L \leq 0 | K(55) \geq 1] = (.955)(.932) = .890$ (only on survival to age 58),

and $P[{}_1L \leq 793 | K(55) \geq 1] = .890 + (.955)(.06) = .94736$ (still not $\geq .95$),

and $P[{}_1L \leq 893 | K(55) \geq 1] = .94736 + .04 = .98736 \geq .95$ . Answer: D

 

13. We can use the recursive relationship
$Var[{}_hL|K(x) \geq h] = [v(b_{h+1} - {}_{h+1}V)]^2\, p_{x+h}\, q_{x+h} + v^2 p_{x+h} Var[{}_{h+1}L|K(x) \geq h+1]$
to find $Var[{}_1L|K(x) \geq 1]$.
Since the policy terminates at time 3, $Var[{}_3L|K(x) \geq 3] = 0$. Then
$Var[{}_2L|K(x) \geq 2] = [v(b_3 - {}_3V)]^2\, p_{x+2}\, q_{x+2} + v^2 p_{x+2} Var[{}_3L|K(x) \geq 3]$
$= [1000v]^2(.5)(.5) = 206,612$ (since ${}_3V = 0$ and $b_3 = 1000$).
Then,
$Var[{}_1L|K(x) \geq 1] = [v(b_2 - {}_2V)]^2\, p_{x+1}\, q_{x+1} + v^2 p_{x+1} Var[{}_2L|K(x) \geq 2]$
$= [v(1000 - 120.833)]^2(.6)(.4) + v^2(.6)(206,612) = 255,762$. Answer: C

14. In order for (30) to die second and within 5 years the death of (35) it must be true that (35) dies first and (30) dies within 5 years after that. This probability is $\int_0^\infty {}_tp_{35}\, \mu_{35}(t)\, {}_tp_{30} \cdot {}_5q_{30+t}$ $dt$.
The integral is set up based on the density of (35)'s death at time $t$, and (30) being alive at the time but dying in the next 5 years. We can write ${}_tp_{30} \cdot {}_5q_{30+t}$ in the form
${}_tp_{30} \cdot {}_5q_{30+t} = {}_tp_{30} \cdot (1 - {}_5p_{30+t}) = {}_tp_{30} - {}_{t+5}p_{30} = {}_tp_{30} - {}_5p_{30} \cdot {}_tp_{35}$.
The integral becomes
$\int_0^\infty {}_tp_{35}\, \mu_{35}(t)\, ({}_tp_{30} - {}_5p_{30} \cdot {}_tp_{35})\, dt$
$= \int_0^\infty {}_tp_{35}\, \mu_{35}(t)\, {}_tp_{30}\, dt \, - \, {}_5p_{30} \int_0^\infty {}_tp_{35}\, \mu_{35}(t) \cdot {}_tp_{35}\, dt$.
The first integral is the probability that 35 will die before 30, which is $1 - b$.
The second integral is $\frac{1}{2}$, because it is the probability that one of two people of equal age 35 will be the first to die. We are also given ${}_5p_{30} = a$.
The overall probability is $1 - b - \frac{1}{2}a$. Answer: E

15. The probability of transform from state 1 to state 1 in the first year is $Q_0^{(1,2)} = .3$.
The probability of transferring from state 1 to state 2 in the second year is
$Q_0^{(1,1)} \cdot Q_1^{(1,2)} = (.6)(.4) = .24$.
The probability of transferring from state 1 to state 2 in the third year is
${}_2Q_0^{(1,1)} \cdot Q_2^{(1,2)} = [(.6)(.4)](.1) = .024$ (we can see from $Q_1$ that since $Q_1^{(1,2)} = 0$, the only way to still be in state 1 at the start of the third year is to stay in state 1 from year 1 to year 2 and the stay from year 2 to year 3).
The actuarial present value of the payments made because of transfer from state 1 to state 2 is
$1000[.3v + .24v^2 + .024v^3] = 405.89$.
The fee $P$ is paid if in state 1. The fee will be paid at the star of the first year.

15 continued
The fee will be paid with probability $Q_0^{(1,1)} = .6$ at the start of the second year.
The fee will be paid with probability ${}_2Q_0^{(1,1)} = .24$ at the start of the third year.
The APV of fees is $P[1 + .6v + .24v^2] = 1.6336P$ .
According to the equivalence principle, we have $1.6336P = 405.89$ , so $P = 248.46$ .
Answer: D

16. In order for Tom to find at least 3 coins in the next two blocks, Tom must find either 1,2 or 2,1 or 2,2 in the next two blocks. The probabilities of these combinations are $Q^{(1,1)}Q^{(1,2)} = (.6)(.3) = .18$ , $Q^{(1,2)}Q^{(2,1)} = (.3)(.5) = .15$ , and $Q^{(1,2)}Q^{(2,2)} = (.3)(.4) = .12$ . The total probability is $.18 + .15 + .12 = .45$ .
Answer: B

17. The probability of getting 100 at the end of the first year is .8.
The probability of getting 100 at the end of the second year is $(.8)(.8) + (.2)(.7) = .78$ (these are the 2 combinations of $N_1N_2$ and $Y_1N_2$ , where $N$ and $Y$ denote the events of no accident and accident).
The probability of getting 100 at the end of the third year is
$(.8)(.8)(.8) + (.8)(.2)(.7) + (.2)(.7)(.8) + (.2)(.3)(.7) = .778$ .
(these are the probabilities of the combinations of $NNN$, $NYN$, $YNN$, $YYN$).
The actuarial present value of the payments is $100[.8v + .78v^2 + .778v^3] = 218.20$ .

The probability of getting $R$ at the end of 3 years is $(.8)^3 = .512$ . The actuarial present value of that payment is $.512Rv^3 = .455166R$ . The two choices are actuarially equivalent if they have the same actuarial present value. Solving for $R$ from $.455166R = 218.20$ results in $R = 479$ .
Answer: D

 

18. From the ultimate column, we have $p_{68} = \frac{7700}{8000}$ , so that $q_{68} = \frac{3}{80}$ .
Then from (ii), we get $4q_{66+2} = 5q_{[66+1]+1}$ , so that $4(\frac{3}{80}) = 5q_{[67]+1}$ ,
from which we get $q_{[67]+1} = .03$ , and then $p_{[67]+1} = .97$ .
Then from $p_{[67]+1} = \frac{\ell_{68}}{\ell_{[67]+1}} = \frac{7700}{\ell_{[67]+1}} = .97$ , we get $\ell_{[67]+1} = 7938.14$ .
We continue in a similar way. From (ii) again we get $4q_{65+2} = 5q_{[66]+1}$ .
But from the ultimate table, we have $q_{67} = 1 - p_{67} = 1 - \frac{8000}{8200} = \frac{200}{8200}$.
Then, $4(\frac{2}{82}) = 5q_{[66]+1}$ so that $q_{[66]+1} = .019512$ .
Then from (i), we get $3q_{[66]+1} = 4q_{[66+1]}$ , so that $q_{[67]} = .014634$ ,
and $p_{[67]} = .985366$ . Since $p_{[67]} = \frac{\ell_{[67]+1}}{\ell_{[67]}} = \frac{7938.14}{\ell_{[67]}} = .985366$ ,
we get $\ell_{[67]} = 8056$ . Answer: C

19. For fully discrete whole life reserves, we have ${}_tV_x = 1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x}$ .
${}_{10}V_{40} = \frac{\ddot{a}_{50}}{\ddot{a}_{40}}$ and ${}_{13}V_{40} = \frac{\ddot{a}_{53}}{\ddot{a}_{40}}$ .
Since ${}_{10}V_{40} = {}_{13}V_{40}$ , it follows that $\ddot{a}_{50} = \ddot{a}_{53} = 10.0$ .
Using the relationship $\ddot{a}_{50} = 1 + vp_{50} + v^2\,{}_2p_{50} + v^3\,{}_3p_{50}\,\ddot{a}_{53}$ , we get
$10 = 1 + vp + v^2p^2 + v^3p^3 \cdot 10$ , where $p$ is the common value of $p_{50}, p_{51}$ and $p_{52}$.
By trial and error we try each of the possible values of $p$. The value $p = .954$ satisfies the equation. Answer: D

20. In order for Derek and A-Rod to survive two years, they must both survive the first year in which the are subject to a single common shock, and they must both survive the second year as independent lives. The probability is $P \times Q \times R$ , where $P$ is the probability that they both survive the first year, $Q$ is the probability that Derek survives the 2nd year, and $R$ is the probability that A-Rod survives the 2nd year. $Q$ and $R$ are both $e^{-.001}$ (they are each subject to the total force of mortality of .001). $P$ is $p^*_D \cdot p^*_A \cdot e^{-.0002}$ (this is the probability that Derek does not die to causes other than the common shock, and A-Rod also doesn't die to causes other than common shock, and the common shock doesn't occur in the first year). Derek' force of mortality due to causes other than common shock is $.001 - .0002 = .0008$, and same for A-Rod. Therefore, $P = e^{-.0008} \cdot e^{-.0008} \cdot e^{-.0002} = e^{-.0018}$. The total probability we are looking for is $e^{-.0018} \cdot e^{-.001} \cdot e^{-.001} = .9962$ . Answer: C

 

21. The original model is a DeMoivre model. Survival under the new model is based on a Generalized DeMoivre model. The new model has a new $\alpha$, but the same $\omega$.
(ii) tells us that $\frac{\omega-30}{\alpha+1} = \frac{4}{3} \cdot \frac{\omega-30}{2}$ . It follows that $\alpha = \frac{1}{2}$ .
(iii) tells us that $\frac{\omega-60}{\alpha+1} = 20$ , so that $\omega - 60 = 20(\frac{1}{2}+1) = 30$ , and $\omega = 90.$
Under the original DeMoivre model, we have $\mathring{e}_{70} = \frac{90-70}{2} = 10.$ Answer: B

22. $Z = 1000e^{-.1T}$ if death occurs with $T \leq 10$ years, and $Z = 2500e^{-.1T}$ if $T > 10$.
$1000e^{-.1t} > 700$ if $-.1t > ln(.7)$ , or equivalently, $t < \frac{ln(.7)}{-.1} = 3.57$ .
$2500e^{-.1t} > 700$ if $-.1t > ln(.28)$, or equivalently, $t < \frac{ln(.28)}{-.1} = 12.73$ , but in this case, we must also have $t > 10$ (for the benefit to be 2500).
The total probability is the combination of $P(T < 3.57)$ and $P(10 < T < 12.73)$ .
This is ${}_{3.57}q_{40} + {}_{10|2.73}q_{40} = \frac{3.57}{100-40} + \frac{2.73}{100-40} = .105$ . Answer: C

23. ${}_{1|1}q_x^{(2)} = p_x^{(\tau)} \cdot q_{x+1}^{(2)}$
For a 2 decrement table, $p_x^{(\tau)} = p_x'^{(1)} \cdot p_x'^{(2)}$ .
We are given $q_x'^{(1)} = .1$, so that $p_x'^{(1)} = .9$ .
From constant force $\mu_x^{(2)} = .2$, we get $p_x'^{(2)} = e^{-.2}$ , so that $p_x^{(\tau)} = .9e^{-.2} = .7369$ .
From $q_{x+1}'^{(2)} = .25$ we get $p_{x+1}'^{(2)} = .75$ , and then from constant force of decrement we get $\mu_{x+1}^{(2)} = -ln(.75) = .2877$ , and $\mu_{x+1}^{(\tau)} = .15 + .2877 = .4377$ .
Then, also from constant force of decrement, $q_{x+1}^{(2)} = \frac{\mu_x^{(2)}}{\mu_x^{(\tau)}} \cdot q_x^{(\tau)}$ .
From constant force ,we have $p_{x+1}^{(\tau)} = e^{-\mu_{x+1}^{(\tau)}} = e^{-.4377} = .6455$ . so that $q_{x+1}^{(\tau)} = .3545,$
and then $q_{x+1}^{(2)} = \frac{.2877}{.4377} \cdot (.3545) = .233$ .
Finally, ${}_{1|1}q_x^{(2)} = p_x^{(\tau)} \cdot q_{x+1} = (.7369)(.233) = .172$ . Answer: B

24. $\ddot{a}_{75:\overline{3|}} = 1 + vp_{75} + v^2\,{}_2p_{75}$ .
From (i) we get ${}_np_{75} = e^{-\int_{75}^{75+n}\mu(t)\,dt} = e^{-.01[(75+n)^{1.2}-75^{1.2}]}$.
Therefore, $p_{75} = e^{-.01[76^{1.2}-75^{1.2}]} = .9719$ and
${}_2p_{75} = e^{-.01[77^{1.2}-75^{1.2}]} = .9445$ .
The APV of the annuity is $1 + \frac{.9719}{1.11} + \frac{.9445}{(1.11)^2} = 2.64$ . Answer: A

 

25. The expected number of trains that will arrive between 7:00 AM and 7:25AM. is
$\int_0^{25} \lambda(t)\, dt = \int_0^{10} (.05)\, dt \,+\, \int_{10}^{20} \left(\frac{t}{200}\right) dt \,+ \int_{20}^{25} (.1)\, dt = .5 + .75 + .5 = 1.75$ .
The probability of exactly four trains in the time interval is the Poisson probability
$\frac{e^{-1.75}(1.75)^4}{4!} = .070$ . Answer: B

26. For a Poisson process with rate $\lambda$ per unit time, the time of the $n$-th event, $S_n$, has a gamma distribution with mean $\frac{n}{\lambda}$ and variance $\frac{n}{\lambda^2}$ . In this problem, $\lambda = 4$ and $T = S_{289}$ , so $T$ has mean $\frac{289}{4} = 72.25$ and variance $\frac{289}{16} = 18.0625$ . Applying the normal approximation to $T$, we get $P(T > 68) = P\left(\frac{T - 72.25}{\sqrt{18.0625}} > \frac{68 - 72.25}{\sqrt{18.0625}}\right) = 1 - \Phi(-1) = .8413.$ Answer: A

27. $E[Z] = \int_0^{\infty} e^{-t} e^{-\delta t}\, e^{-\mu t} \cdot \mu\, dt = \frac{\mu}{1+\delta+\mu} = .03636$ .
Since $\delta = .06$ , it follows that $\mu = .04$ .
Then, $E[Z^2] = \int_0^{\infty} e^{-2t} e^{-2\delta t}\, e^{-\mu t} \cdot \mu\, dt = \frac{\mu}{2+2\delta+\mu} = .01852$ .
$Var[Z] = .01852 - (.03636)^2 = .0172$ , Answer: A

28. The recursive relationship for assets shares is

$$\left[{}_{h-1}AS + G(1 - c_{h-1}) - e_{h-1}\right](1+i) - bq^{(1)}_{x+h-1} - {}_hCV\, q^{(2)}_{x+h-1} = p^{(\tau)}_{x+h-1}\, {}_hAS \; .$$

Using this, we have

$$\left[{}_{15}AS + G(1 - c_{15}) - e_{15}\right](1+i) - bq^{(1)}_{40+15} - {}_{16}CV\, q^{(2)}_{40+15} = p^{(\tau)}_{40+15}\, {}_{16}AS \; ,$$

which becomes
$[1150 + 90(.95)](1.08) - 10{,}000(.004) - {}_{16}CV(.05) = (.946)(1320)$ .
Solving for ${}_{16}CV$ results in ${}_{16}CV = 912.40$ . Answer: C

29. The equivalence principle equation is $Q = 200\, {}_{30|}\ddot{a}_{30} + Q A_{\overset{1}{30:\overline{30|}}}$ .
${}_{30|}\ddot{a}_{30} = {}_{20}E_{30} \cdot {}_{10}E_{50} \cdot \ddot{a}_{60} = (.29374)(.51081)(11.1454) = 1.672$ .
$A_{30} = A_{\overset{1}{30:\overline{30|}}} + {}_{20}E_{30} \cdot {}_{10}E_{50} \cdot A_{60}$ , so that
$A_{\overset{1}{30:\overline{30|}}} = .10238 - (.29374)(.51081)(.36913) = .04699$ .
Solving for $Q$ results in $Q = \frac{200(1.672)}{1 - .04699} = 351$ . Answer: A

30. The benefit premium is $Q$. The equivalence principle equation is
$Q\int_0^\infty e^{-\delta t}\,{}_tp_x^{(\tau)}\,dt = 3\cdot\int_0^\infty e^{-\delta t}\,{}_tp_x^{(\tau)}\mu_x^{(1)}(t)\,dt + \int_0^\infty e^{-\delta t}\,{}_tp_x^{(\tau)}\mu_x^{(2)}(t)\,dt$ .
Since $\mu_x^{(1)}(t) = .02$ and $\mu_x^{(2)}(t) = .04$ , it follows that $\mu_x^{(\tau)}(t) = .06$ ,
and ${}_tp_x^{(\tau)} = e^{-.06t}$ . The equation becomes $Q\cdot\frac{1}{\delta+06} = 3\cdot\frac{.02}{\delta+.06} + \frac{.04}{\delta+.06}$
which becomes $Q = 3(.02) + .04 = .10$ . Answer: D

 